

NISH  
Chapter 2

Complex Numbers

$$i = \sqrt{-1}$$

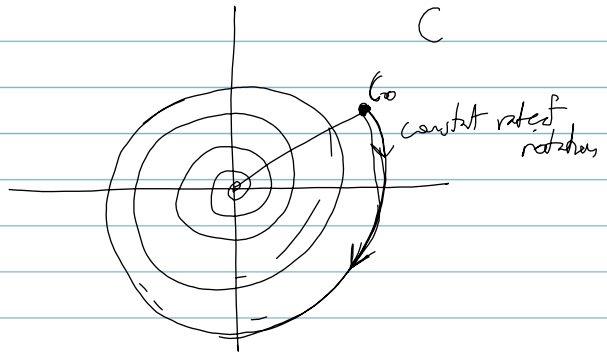
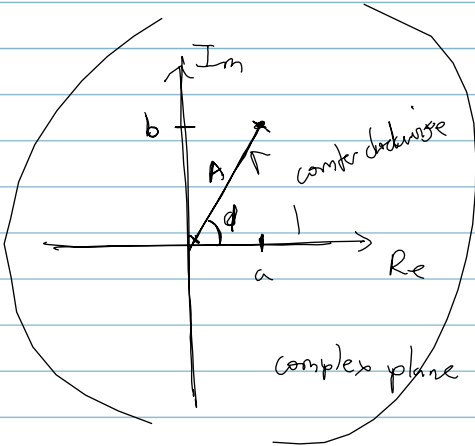
$$c = a + bi$$

Real part  
Imaginary part

$$= A e^{i\phi}$$

magnitude  
phase "phase"

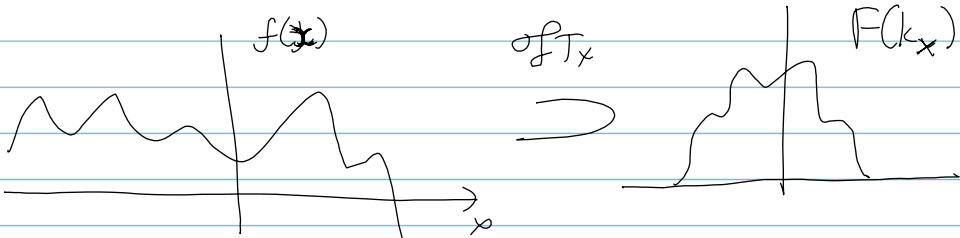
$$e^{i\phi} = \cos \phi + i \sin \phi$$



Laplace transform  
decomposition

$$c(t) = \underline{C_0 e^{j\omega_0 t - R_2 t}}$$

## Continuous Fourier Transform



$$f(x) \Rightarrow F(k_x)$$

$\swarrow$   $\searrow$   
 $c_m$   $c_{m-1}$   
 $\omega, \sigma, \nu$

$$F(k_x) = \int_{-\infty}^{\infty} \underline{f(x)} e^{-i 2\pi k_x x} dx$$
$$= \int_{-\infty}^{\infty} f(x) \cos 2\pi k_x x dx - i \int_{-\infty}^{\infty} f(x) \sin 2\pi k_x x dx$$

if  $f(x)$  is real valued

$F(k_x)$  has Hermitian symmetry

$$F(-k_x) = F(k_x)^*$$

invertible

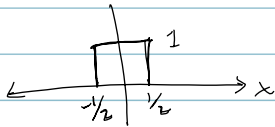
$$f(x) = \int_{-\infty}^{\infty} F(k_x) e^{+i 2\pi k_x x} dk_x$$

$$f(x, y), f(x, y, z), f(x, y, z, t)$$

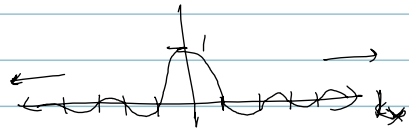
$$F(k_x, k_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) e^{-j2\pi(k_x x + k_y y)} dx dy$$

[Important Fourier Transform Pairs]

$$\Pi(x) \triangleq \begin{cases} 1 & |x| < 1/2 \\ 0 & \text{else} \end{cases}$$



$$\text{sinc}(k_x) \triangleq \frac{\sin \pi k_x}{\pi k_x}$$



$\text{sinc}(x)$

$\Pi(k_x)$

$$\delta(x) \triangleq \lim_{\Delta \rightarrow 0} \frac{1}{\Delta} \Pi\left(\frac{x}{\Delta}\right)$$



$\underline{1}$

$\underline{1}$

$\delta(k_x)$

$$e^{-\pi x^2}$$

$$e^{-\pi kx^2}$$

$$f(x-x_0)$$

$$e^{-j2\pi kx_0 x}$$

$$f(x-x_0)$$

$$F(kx) e^{-j2\pi kx_0 x}$$

linear phase

$$f(ax)$$

$$\frac{1}{|a|} F\left(\frac{kx}{a}\right)$$

$$f(x) \cdot g(x)$$

$$F(kx) * G(kx)$$

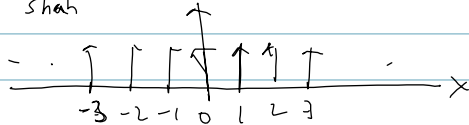
$$f(x) * g(x)$$

$$F(kx) G(kx)$$

Sampling

$$\text{III}(x) \triangleq \sum_{k=-\infty}^{\infty} \delta(x-kc)$$

"shah"

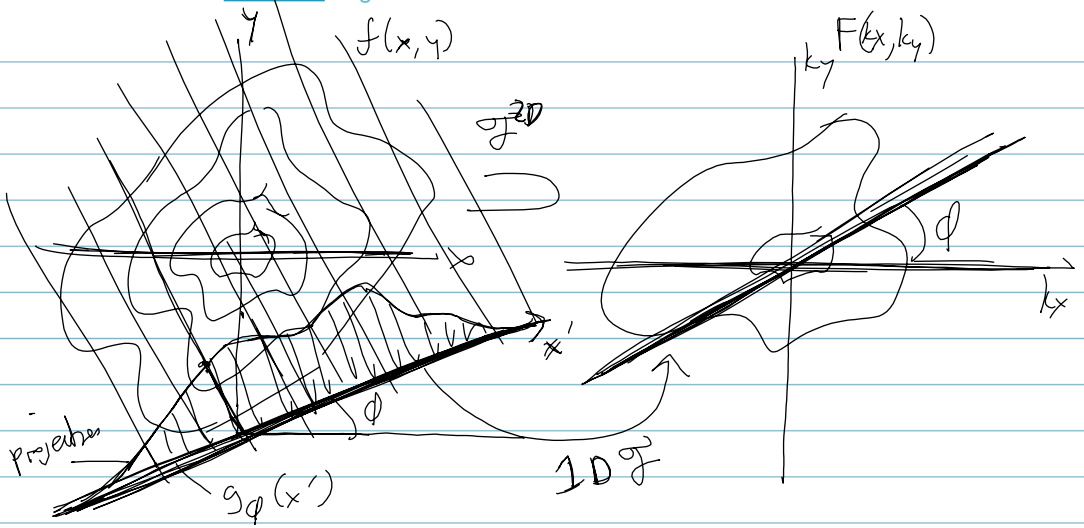


$$\text{III}(kx)$$

$$f(x,y) = f_x(x) f_y(y)$$

↑  
separable

$$\mathcal{F}^{1D}(f_x(x)) \mathcal{F}^{1D}(f_y(y))$$



central section theorem

consider  $\phi = 0$

$$g_0(x) = \int_{-\infty}^{\infty} f(x, y) dy$$

$$\mathcal{F}^{1D} \{ g_0(x) \} = \int_{-\infty}^{\infty} \underbrace{\int_{-\infty}^{\infty} f(x, y) dy}_{g_0(x)} e^{-i 2\pi k_x x} dx$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) e^{-i 2\pi (k_x x + \phi y)} dx dy$$

↑  
dxdy

$$= F(k_x, k_y) \Big|_{k_y = 0}$$