

# MAGNETIC RESONANCE IMAGING

Physics

"Nuclear Magnetic Resonance"  
discovered independently by Bloch & Purcell

"Classical Descriptions" (full desc. requires QM)

atoms w/ odd # protons and/or neutrons

Spin angular momentum  $\vec{S} = \hbar \vec{I}$   
↑ ← spin quantum number  
 $\frac{\text{Planck's const}}{2\pi}$



magnetic dipole moment  $\vec{\mu} = \gamma \vec{S}$   
← gyromagnetic ratio

charged spinning sphere =  $\gamma \hbar \vec{I}$

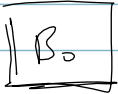


Atoms?  $^1\text{H}$ ,  $^{31}\text{P}$ ,  $^{23}\text{Na}$ ,  $^{13}\text{C}$   
most abundant "Spins"

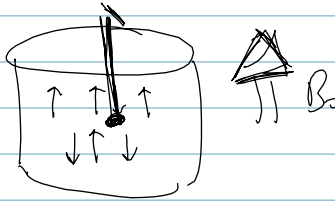
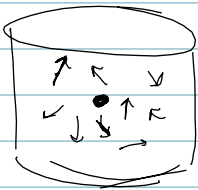
# Magnetic Fields



- $B_0$  - static, v. strong
- $B_1$  - radio frequency
- $G_{x,y,z}$  - spatial encoding



Polarization: achieves macroscopic magnetization



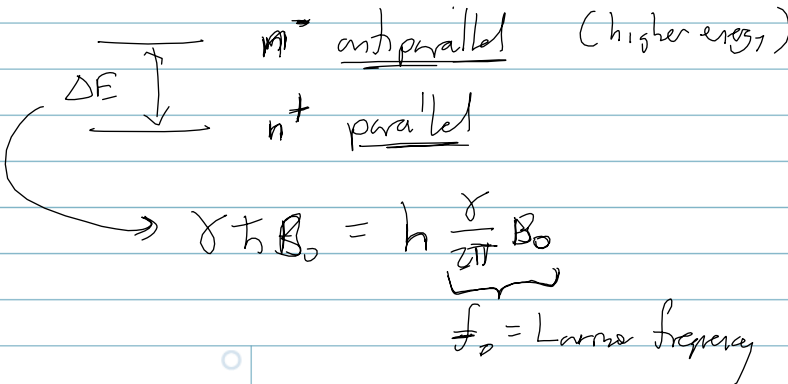
$$\begin{aligned} \text{energy } \vec{\mu} \text{ in } \vec{B}_0 &= -\vec{\mu} \cdot \vec{B} = -\mu_z B_0 \\ &= \pm \frac{\gamma \hbar B_0}{2} \end{aligned}$$

$$= -\gamma \hbar I B_0$$

↑

$$I = \pm \frac{1}{2}$$

two energy states



Boltzmann distribution  $\swarrow$  energy diff

$$\frac{n^-}{n^+} = e^{-\frac{\Delta E}{kT}}$$

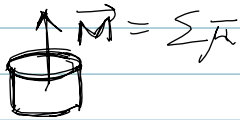
$\nwarrow$  absolute temperature  
 $\swarrow$  Boltzmann const

@  $^1\text{H}$ , Room Temp, 1.5 T

$$\frac{n^-}{n^+} \approx 0.999993$$

"weak" polarization

basis for almost all MRI.



Establishes Resonance Condition

@ equilibrium  $\vec{M} \parallel \vec{B}_0$

If  $\vec{M} \not\parallel \vec{B}_0$  there is an induced torque

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B}$$

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⊙ out of board  
 $\vec{M} \times \vec{B}$

⊙ is preserved

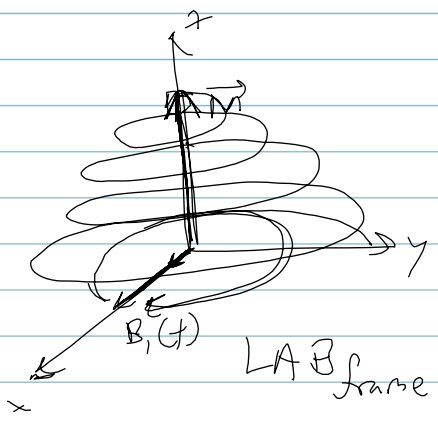


frequency is  $\gamma |B| = f_0$

$$\frac{\gamma}{2\pi} = 42.58 \text{ MHz/T}$$

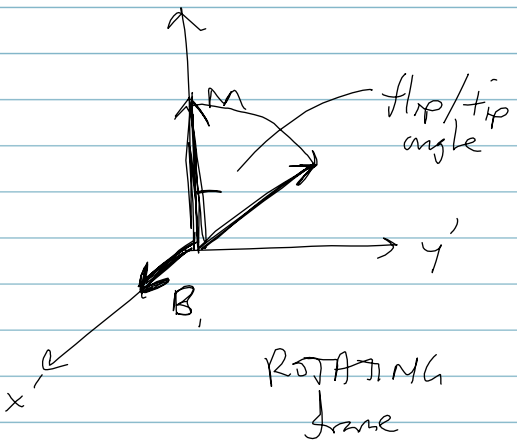
$B_0$  provides polarization & resonance conditions

$B_1$  radiofrequency  $\perp B_0$  to excite nuclei to states



apply rotates  $B_1(t)$  in the transverse plane at  $f_0$

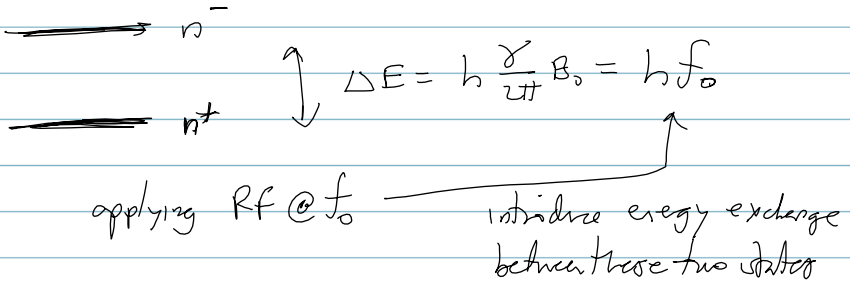
perturb nuclei



$x', y'$  rotates @  $f_0$

$$\frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma(\vec{B}_{eff})$$

another look:



yet another look:

