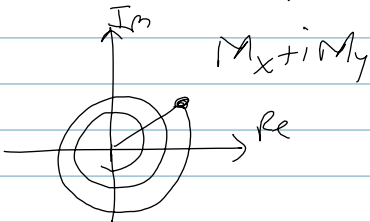


## Another derivation of the Signal Equation

$$S_r(t) = \int m(x, y, t) dx dy \quad \left. \begin{array}{l} \text{slice} \\ \text{(plane)} \end{array} \right\} \text{ignore } T_z$$
$$= \int_{x, y} m(x, y) e^{-i \phi(x, y, t)} dx dy$$



$$\phi(x, y, t) ??$$

$$\omega(t) = \frac{d\phi(t)}{dt}$$

$$\therefore \phi(t) = \int_0^t \omega(\tau) d\tau$$

$$\omega(x, y, t) = \gamma B_0(x, y, t)$$

$\underbrace{\hspace{10em}}$  includes  $B_0$  &  $G$

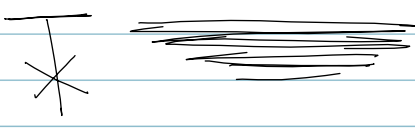
$$= \gamma \left( B_0 + \underbrace{G_x(t)x + G_y(t)y}_{\text{control}} \right)$$

$$d(x, y, t) = \int_0^t B_0 + G_x(\tau)x + G_y(\tau)y \, d\tau$$

$$= \omega_0 t + 2\pi \underbrace{\left( \frac{\delta}{2\pi} \int_0^t G_x(\tau) d\tau \right)}_{k_x(t)} x + 2\pi \underbrace{\left( \frac{\delta}{2\pi} \int_0^t G_y(\tau) d\tau \right)}_{k_y(t)} y$$

$$= \omega_0 t + 2\pi (k_x(t)x + k_y(t)y)$$

$$S_r(t) = \iint_{xy} m(x, y) e^{-i\omega_0 t - i2\pi(k_x(t)x + k_y(t)y)} \, dx dy$$

baseband 

$$s(t) = S_r(t) e^{i\omega_0 t} = \mathcal{M}(k_x(t), k_y(t))$$

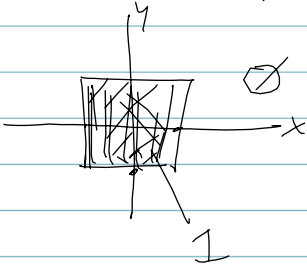
### key points

1) using  $G_x$  &  $G_y$  to control the instantaneous frequency at different  $x, y$  positions  $\rightarrow$  imparts a space dependent phase

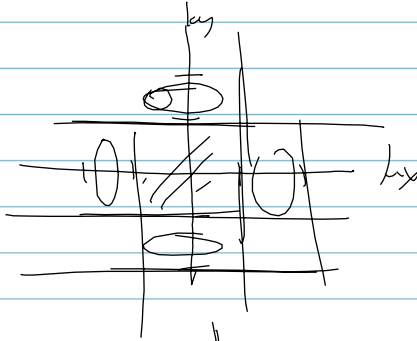
2) linear gradients  $\rightarrow$  impart a linear phase variation

3) phase  $\phi = 2\pi \underbrace{k_x(t)x}_{\text{phase per unit distance}} + \dots$  cycles per unit distance

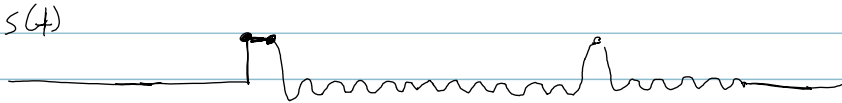
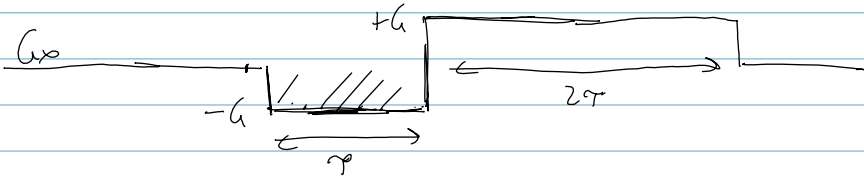
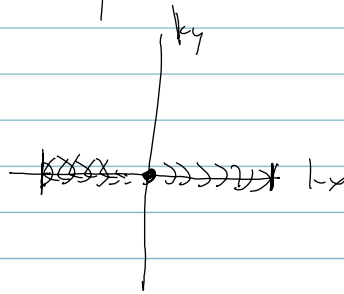
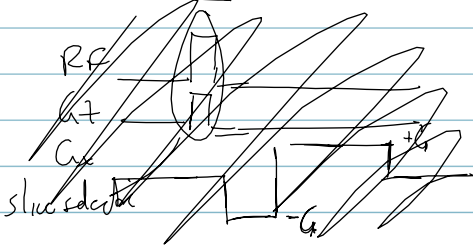
consider  $m(x,y) = \text{rect}(x,y)$



$$M(k_x, k_y) = \text{sinc}(k_x) \text{sinc}(k_y)$$



Pulse sequence



$$\left( -\frac{\delta}{2T} G_x T, \frac{\delta}{2T} G_x T \right) \text{ range covered along } k_x$$

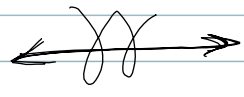
**BREAK**

complex demodulation

$$s(t) = s_r(t) e^{+i\omega_0 t}$$

$$s_r(t) = \underline{\alpha(t)} e^{-j(\omega_0 t + \underline{\phi(t)})}$$

physical signal



$$s_p(t) = \text{Re}\{s_r(t)\}$$

$$= \underline{\alpha(t)} \cos(\omega_0 t + \underline{\phi(t)})$$

$$= \underline{\alpha(t) \cos \phi(t) \cos \omega_0 t} - \underline{\alpha(t) \sin \phi(t) \sin \omega_0 t}$$

want baseband signal

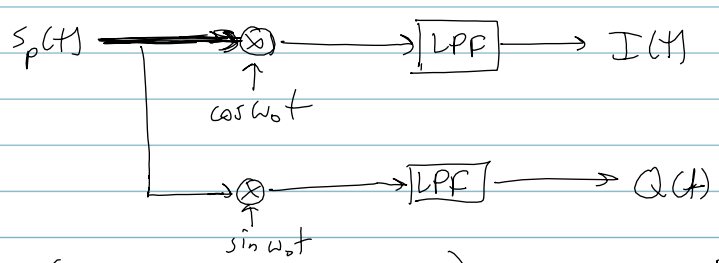
orthogonal & in quadrature

$$s(t) = \alpha(t) e^{-i\phi(t)}$$

$$= \underline{\alpha(t) \cos \phi(t)} - i \underline{\alpha(t) \sin \phi(t)}$$

I(t)                      Q(t)

Quadrature Phase Sensitive Detector (QPSD)



this is how it is done!!!

$$\left( \alpha(t) \cos(\omega_0 t + \phi(t)) \cos \omega_0 t + \frac{\pi}{2} \right) * h(t) \Rightarrow \alpha(t) \cos \phi(t)$$

LPP