

## Excitation Chapter 6

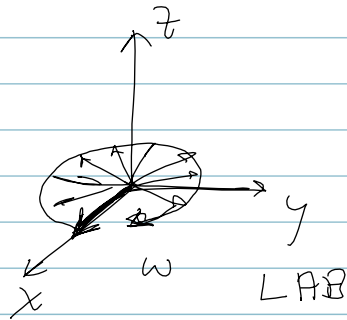
1) non-selective (G off)

2) selective excitation (G on)

non-selective excitation

RF field @ carrier freq  $\omega$

envelope  $B_1(t)$



drive  $\omega = \omega_0$

$$B_1(t) (\cos \omega t \hat{i} - \sin \omega t \hat{j})$$

$$\text{Bloch Eq'n : } \frac{d\vec{m}}{dt} = \vec{m} \times \gamma \vec{B}$$

$$\vec{B} = B_0 \hat{k} + B_1(t) (\cos \omega t \hat{i} - \sin \omega t \hat{j})$$

Rotating Frame rotates around z axis  
with frequency  $\omega$

$$\vec{M}_{rot} = \begin{bmatrix} M_x' \\ M_y' \\ M_z \end{bmatrix} \quad \vec{B}_{rot} = \begin{bmatrix} B_x' \\ B_y' \\ B_z \end{bmatrix}$$

$$\vec{M}_{lab} = R_z(\omega t) \vec{M}_{rot}$$

$$\vec{B}_{lab} = R_z(\omega t) \vec{B}_{rot}$$

Simplify  $B_{lab}$  in RA Frame (derivation in  
Ch 6 App I)

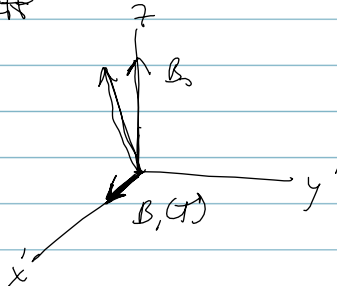
$$\frac{d\vec{m}_{rot}}{dt} = \vec{m}_{rot} \times \left[ \gamma \vec{B}_{rot} - \omega \hat{k} \right] \quad *$$

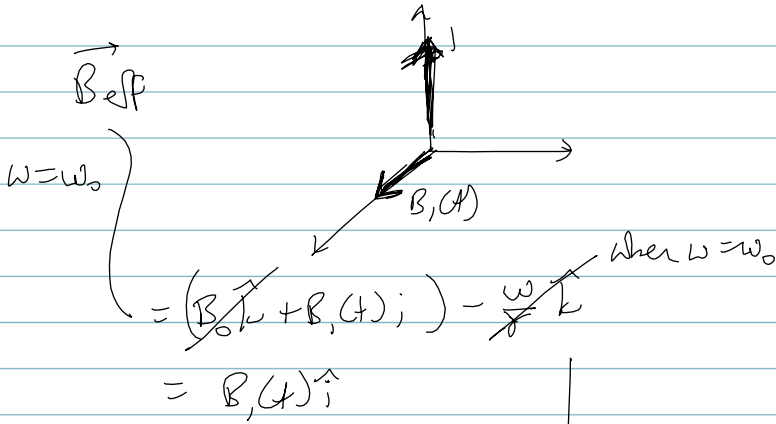
$$\vec{B}_{eff} \triangleq \vec{B}_{rot} - \frac{\omega}{\gamma} \hat{k}$$

$$\frac{d\vec{m}_{rot}}{dt} = \vec{m}_{rot} \times \gamma \vec{B}_{eff}$$

$$\vec{B}_{rot} = B_0 \hat{k} + B_1(t) \hat{i}$$

last page





freq of rotation

$$\gamma |\vec{B}_{eff}| = \gamma B_1(t) = \omega(t)$$

$$\Theta(t) = \int_0^t \omega(t) dt$$

"flip angle"

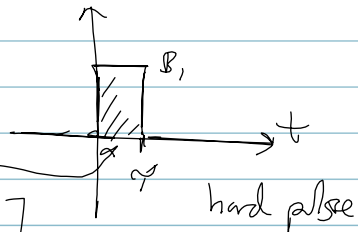
"tip angle"

$$B_1(t) = \begin{cases} B, & 0 < t < \tau \\ 0 & \text{else} \end{cases}$$

$$\omega_1 = \gamma B_1$$

$$\Theta = \gamma B_1 \tau$$

$$= \gamma \left[ \text{area under } B_1(t) \right]$$

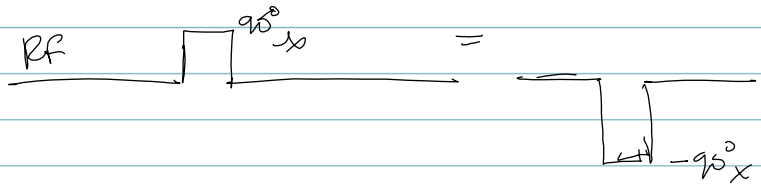
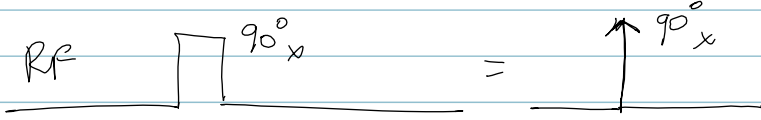


Terms: 1) tip angle  $\Theta$

2) axis of rotation

$\vec{B}_{eff}$  can be oriented along any direction  $\vec{u}$

## Pulse Sequence diagrams



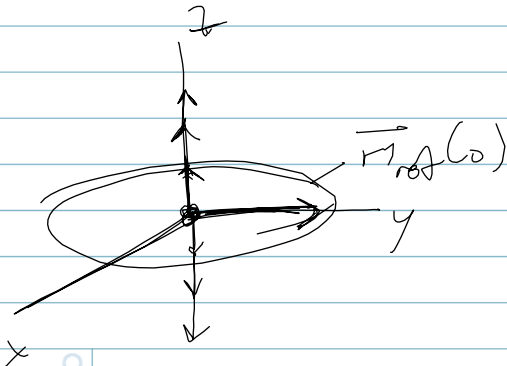
Example #2 : , adiabatic , no RF

$$\vec{B} = (B_0 + G_x x) \hat{k}$$

rot frame -  $\omega_0$

$$\vec{B}_{\text{eff}} = \vec{B}$$

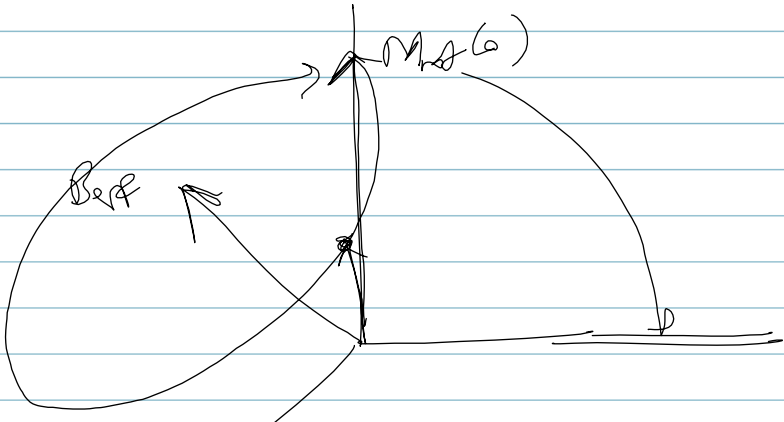
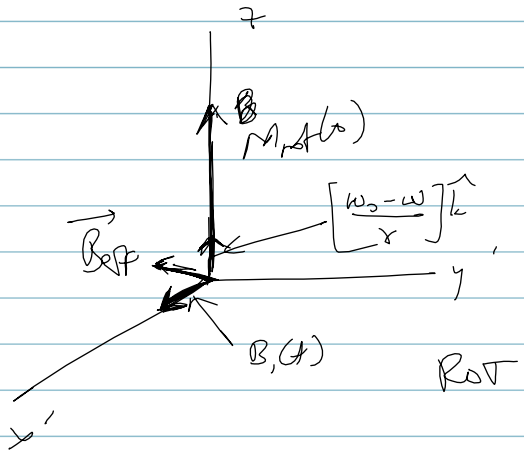
$$\vec{B}_{\text{eff}} = [B_0 + G_x x] \hat{k} - \frac{\omega_0}{\gamma} \hat{k} = G_x x \hat{k}$$



G aff  
B, on

what if  $\omega \neq \omega_0$

$$B_{eff} = \left[ \frac{\omega_0 - \omega}{\gamma} \right] \hat{k} + \underline{B_1(t)}$$

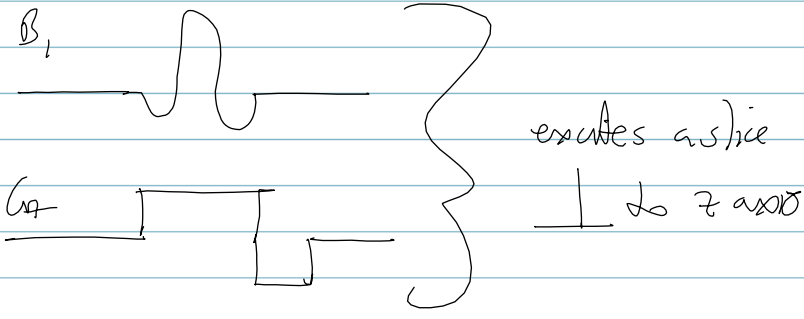


will be the basis for  
selective excitation

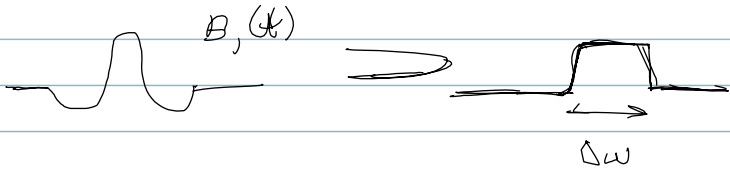
$\rightarrow$  G creates ~~the~~  $\Delta\omega$  mismatch at  
all positions except for a slice of interest

selected  
excitation  
preview

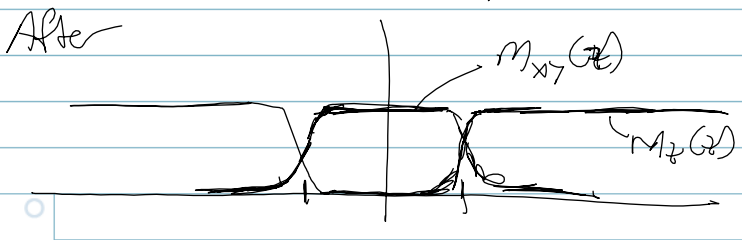
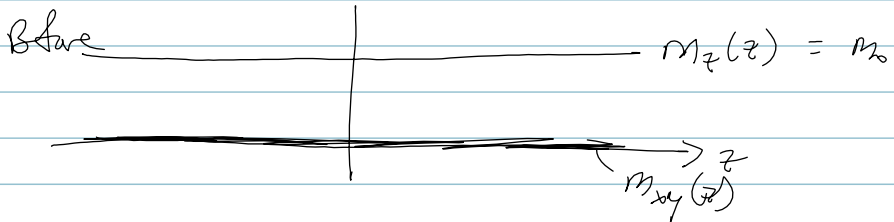
$B_1 + G_2$  simultaneous



slice profile  $M_{xy}(z)$  depends on the  $\int \{B_1(t)\}$



slice profile plots  $M_{xy}^{(z)}$ ,  $M_z(z)$

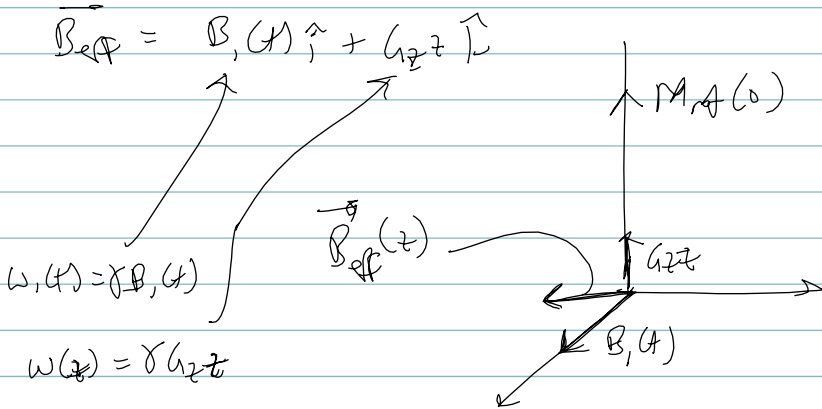


# $B_z(t)$ & $G_z$

$$\vec{B}_{eff} = \begin{bmatrix} B_z(t) \\ \emptyset \\ \frac{\omega_p - \omega}{\gamma} + G_z z \end{bmatrix}$$

let  $\omega = \omega_p$

$$\vec{B}_{eff} = B_z(t) \hat{z} + G_z z \hat{z}$$



- 1)  $z = 0$   $\vec{M}_{eff}$  rotates within the  $y-z$  plane
- 2)  $z$  large  $\vec{B}_{eff}$  almost  $\parallel$  to  $z$ -axis  
 $|G_z z| \gg |B_z(t)|$  no top. rotation
- 3)  $z$  intermediate some smaller/intermediate tipping

$$\frac{d\vec{M}_A}{dt} = \vec{M}_A \times \gamma \vec{B}_{\text{eff}}$$

$$\frac{d\vec{M}_A}{dt} = \begin{bmatrix} 0 & \gamma G_{\text{eff}} & 0 \\ -\gamma G_{\text{eff}} & 0 & \gamma B_0(t) \\ 0 & -\gamma B_0(t) & 0 \end{bmatrix} \vec{M}_A$$

$\swarrow \quad \searrow$   
 $\omega(\mathbb{R}) \quad \quad \omega_1(t)$

suche für transversale magnetisation

$$M_r = M_x' + i M_y'$$

$$\frac{dM_r}{dt} = \frac{dM_x'}{dt} + i \frac{dM_y'}{dt}$$

$$= (\gamma G_{\text{eff}} M_y') + i (-\gamma G_{\text{eff}} M_x' + \gamma B_0(t) M_z')$$

$$= \gamma G_{\text{eff}} (-i M_r') + i \gamma B_0(t) M_z'$$

$$\frac{dM_r}{dt} = -i \omega(\mathbb{R}) M_r' + i \omega_1(t) M_z'$$

$$\frac{dM_z}{dt} = -\omega_1(t) M_y'$$

↓ zwei coupled BE's



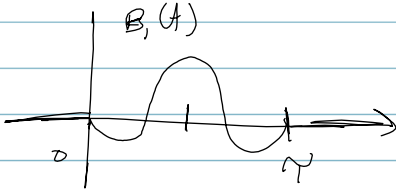
⊗ Small  $T, \rho$  Approximations

$$\theta \leq 30^\circ, \quad \underline{M_z \approx M_0}, \quad \frac{dM_z}{dt} \approx 0$$

$$\frac{dM_r}{dt} = -j\omega(\tau)M_r + \omega_r(t)M_0$$

↓ full solution ch 6, Appx 2

$$M_r(t, \tau) = iM_0 e^{-j\omega(\tau)t} \int_0^t \omega_r(s) e^{+j\omega(\tau)s} ds$$



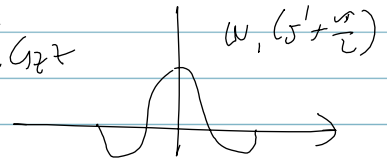
$$\text{let } s' = s - \frac{\tau}{2}$$

$$M_r(\tau, \tau) = iM_0 e^{-j\omega(\tau)\tau} \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} \omega_r(s' + \frac{\tau}{2}) e^{+j\omega(\tau)s'} e^{+j\omega(\tau)\frac{\tau}{2}} ds'$$

$$= iM_0 e^{-j\omega(\tau)\frac{\tau}{2}}$$

$$\int \left\{ \omega_r(t + \frac{\tau}{2}) \right\}$$

$$f = -\frac{\delta}{2\pi} G_{\tau\tau}$$



check it!

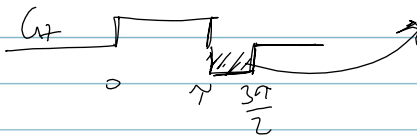
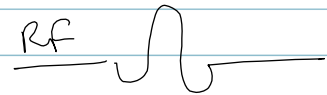
$$z=0$$

$$M_r(\tau, 0) = i M_0 \int \omega, (t) dt = i M_0 \Theta$$

expect  $i M_0 \sin \Theta$

small  $\Theta$  approx  $\sin \Theta \approx \Theta$

Reformms b/c  $e^{-i\omega(t) \tau/2}$



phase shift

$$- \gamma \int_{\tau}^{\frac{3\tau}{2}} = -G_2 \tau dt'$$

$$= \gamma G_2 \tau \frac{\tau}{2}$$

$$= \omega(z) \frac{\tau}{2}$$

$$M_r\left(\frac{3\tau}{2}, z\right) = M_r\left(\tau, z\right) e^{+i\omega(z) \frac{\tau}{2}}$$

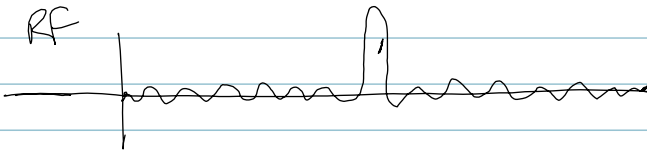
$$= i M_0 \mathcal{F}\left\{\omega, \left(t + \frac{\tau}{2}\right)\right\}$$

$$M_r \left( \frac{3T}{2}, z \right) = M_0 \int \left\{ w, \left( t + \frac{T}{2} \right) \right\}$$

if  $w, \left( t + \frac{T}{2} \right)$  "even"

all excited spins are in-phase

extreme example of refocusing



$$w_{\text{avg}} m(x, y) = \int_{-\frac{\Delta z}{2}}^{\frac{\Delta z}{2}} m(x, y, z) e^{-i \gamma z} dz$$

dephased signal