

# Spatially selective RF excitation using k-space analysis

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## Abstract

This project develops spatially selective RF excitation schemes, based on the excitation k-space principle (Pauly et al., 1987). In particular, we develop various k-space trajectories and weighting schemes, in order to excite cylindrical, spherical, and cubical volumes. The excitation pulses are inherently refocused, since the trajectories end at the origin of k-space. The analysis is based on a small-tip-angle approximation, but behaves well for larger tip angles.

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## 1 Introduction

Magnetic resonance imaging reconstruction has been successfully analyzed using the representation of k-space, and frequency sampling. Pauly et al. (1987) used the same idea to analyze spatially selective RF excitation schemes. The excitation k-space is an elegant tool, and greatly simplifies the design of efficient excitation schemes that target specific areas of the body (Morrell and Macovski, 1997; Bornert and Aldefeld, 1998).

Using the small-tip approximation, we can find an integral expression for the transverse magnetization produced by a selective excitation pulse. This expression may be interpreted as scanning a path in a spatial frequency space, or k-space:

$$M_{xy}(\mathbf{x}) = i\gamma M_o \int_{\mathbf{K}} W(\mathbf{k}) S(\mathbf{k}) e^{i\mathbf{x}\mathbf{k}} d\mathbf{k} \quad (1)$$

where  $W(\mathbf{k})$  is a weighting function, defined by:

$$W(\mathbf{k}(t)) = \frac{B_1(t)}{\left| \frac{g}{2\pi} \mathbf{G}(t) \right|} \quad (2)$$

Further,  $S\mathbf{k}$  is a sampling structure, determining the area and the density of k-space representation:

$$S(\mathbf{k}) = \int_0^T \{^3\delta(\mathbf{k}(t) - \mathbf{k}) |\dot{\mathbf{k}}(t)|\} dt \quad (3)$$

In this brief report, we present pulse sequences to excite a cylindrical volume, a gaussian-weighted sphere, and a cube. We also investigate the robustness of the excitation schemes to off-resonance.

## 2 Exciting Cylindrical Volume

In order to excite a cylindrical volume in the MR scanner, we choose a spiral trajectory with 20 cycles, shown in Fig. 2. The gradients required to achieve this trajectory (Fig. 2) are estimated using Eq. 4:

$$\mathbf{k}(t) = -\frac{\gamma}{2\pi} \int_t^T \mathbf{G}(s) ds \quad (4)$$

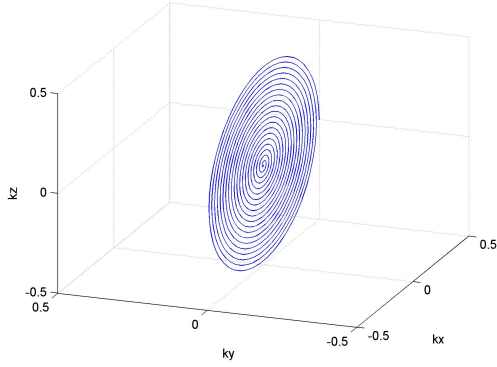


Fig 1: Spiral trajectory is used to excite a cylinder.

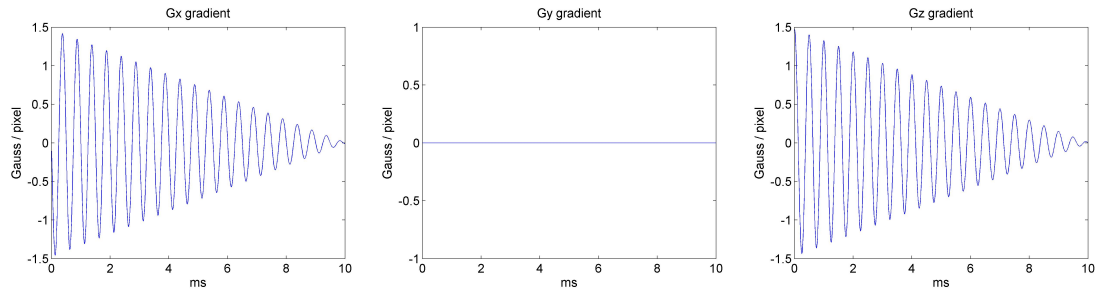


Fig 2: Gx, Gy, and Gz gradients for the spiral k-space trajectory.

In order to excite a cylinder in the spatial domain, we need to weight with a jinc function in the spatial frequency domain:

$$W(\mathbf{k}) = \alpha \cdot \text{jinc}(\beta \sqrt{k_x^2 + k_z^2}) \quad (5)$$

where  $\alpha$  scales the tip angle, and  $\beta$  determines the spatial resolution of the selective volume.

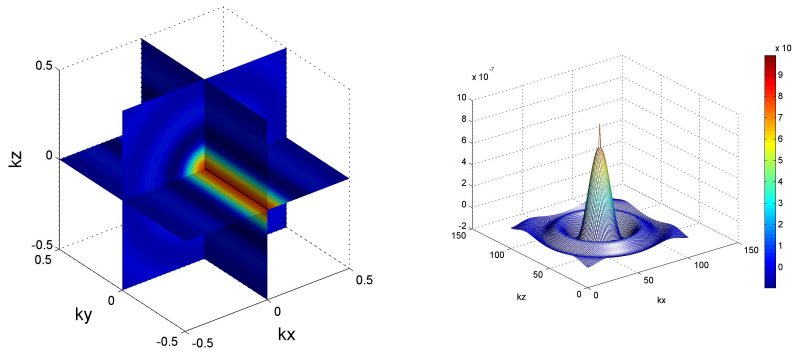


Fig 3: Left: Spatial frequency weighting using a jinc function; Right: Jinc function at  $k_z = 0$ .

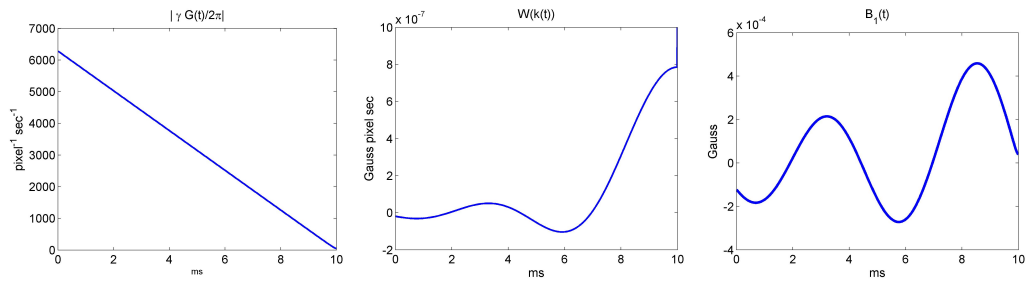


Fig 4: Estimation of RF field using Eq. 2. Left:  $|\frac{\gamma}{2\pi} \mathbf{G}(t)|$  for the cylindrical trajectory; Center: weighting function for the cylindrical trajectory; Right: RF field

We simulated the pulse sequence to selectively excite a cylinder, and the results are given in Fig. 5 and Fig. 6.

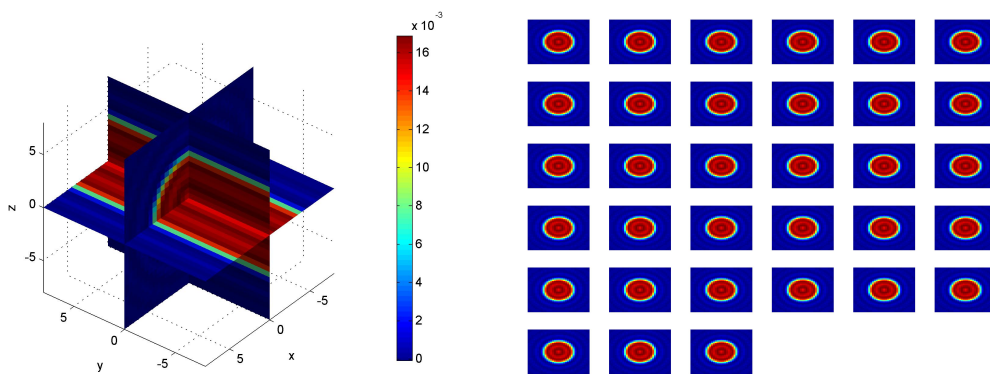


Fig 5: Simulation results for the cylinder excitation. The excitation region is in great accordance with the theoretical prediction.

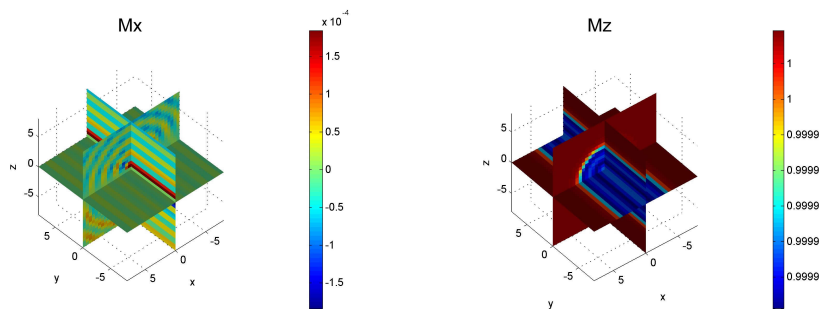


Fig 6: Mx and Mz components for the cylinder excitation. Mx is nearly zero, as expected. Conversely, the Mz component is only small in the cylindrical region, and remains unaffected elsewhere.

### 3 Exciting a Gaussian-Weighted Sphere

In order to limit the object domain excitation region in all 3 dimensions, we need to apply 3 gradients (G<sub>x</sub>, G<sub>y</sub>, and G<sub>z</sub>), and develop a 3D trajectory. This trajectory must sufficiently sample the k-space in x,y, and z dimension.

We begin by designing a spiral trajectory with  $n = 50$  cycles. Then, we slowly rotate each cycle using Rx and Rz rotating matrices, so that the trajectory traverses all 3 dimensions, as shown in Fig. 7.

$$\begin{aligned} k_x(t) &= A(1 - t/T) \cos(2\pi nt/T); \\ k_y(t) &= 0; \\ k_z(t) &= A(1 - t/T) \sin(2\pi nt/T); \end{aligned} \quad (6)$$

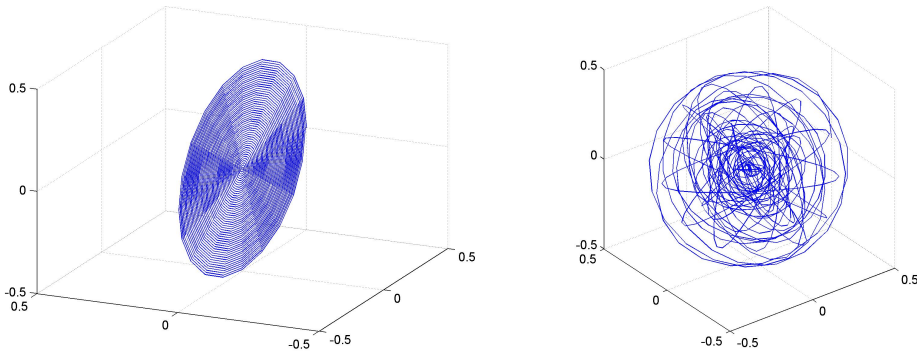


Fig. 7: Left: Spiral trajectory used to generate the 3D trajectory; Right: 3D trajectory used to weight the excitation space.

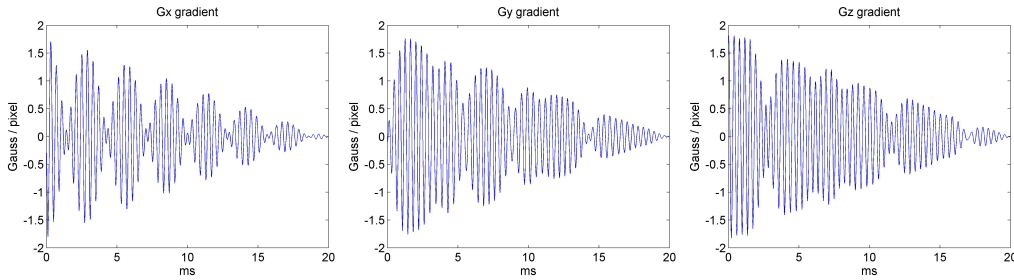


Fig 8: G<sub>x</sub>, G<sub>y</sub>, and G<sub>z</sub> gradients for the 3D k-space trajectory.

In order to excite a gaussian-weighted sphere in the spatial domain, we need to weight with a gaussian-weighted function in the spatial frequency domain:

$$W(\mathbf{k}) = \alpha \cdot e^{-\beta^2(k_x^2 + k_y^2 + k_z^2)/A^2} \quad (7)$$

where  $\alpha$  scales the tip angle, and  $\beta$  determines the spatial resolution of the selective volume.

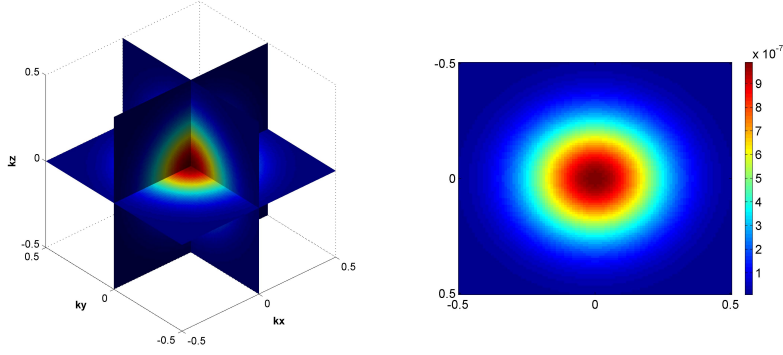


Fig 9: Left: Spatial frequency weighting using a Gaussian-weighted function (Eq. 7) ; Right: Profile of the function at  $k_z = 0$ .

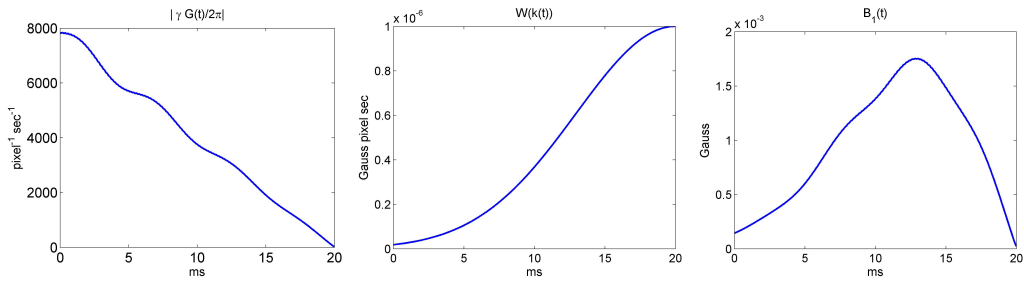


Fig 10: Estimation of RF field using Eq. 2. Left:  $|\frac{\gamma}{2\pi} \mathbf{G}(t)|$  for the 3D trajectory; Center: weighting function for the cylindrical trajectory; Right: RF field

We simulated the pulse sequence to selectively excite a gaussian-weighted sphere, and the results are given in Fig. 11 and Fig. 12.

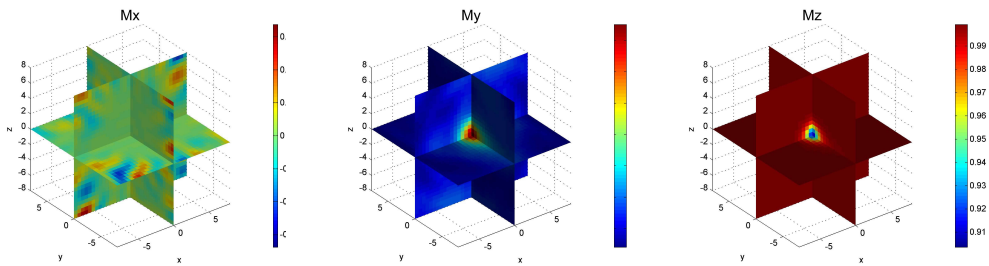


Fig 11:  $M_x$ ,  $M_y$ , and  $M_z$  components for the Gaussian-sphere excitation.  $M_x$  is nearly zero, as expected.  $M_y$  is in great accordance with the theoretical results. The  $M_z$  component is only small in the central region, and remains unaffected elsewhere.

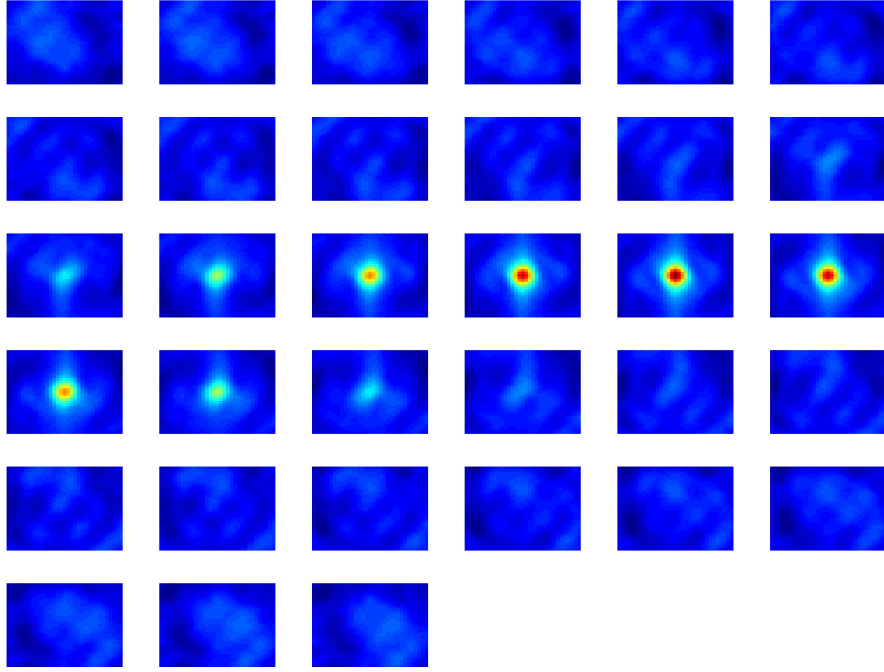


Fig 12: Z-profile of My magnetization component. Only the central region is excited.

#### 4 Exciting a Cube

In order to excite a cube in the spatial domain, we need to weight with a sinc function in the spatial frequency domain:

$$W(\mathbf{k}) = \alpha \cdot \text{sinc}(\beta k_x) \text{sinc}(\beta k_y) \text{sinc}(\beta k_z); \quad (8)$$

where  $\alpha = 10^{-6}$  scales the tip angle, and  $\beta = 8$  determines the spatial resolution of the selective volume.

Unfortunately, this excitation scheme is only of theoretical interest. The required RF pulse (Fig. 13) is impossible to generate with current hardware limitations. In order to excite a cube, we need to implement a different trajectory, which will lead to a more reasonable RF pulse.

We simulated the pulse sequence to selectively excite a cube, and the results are given in Fig. 15.

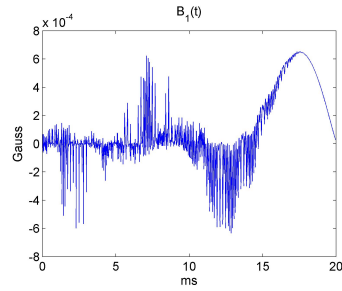


Fig 13: RF pulse for excitation of a cube. The pulse is not realizable with current hardware limitations.

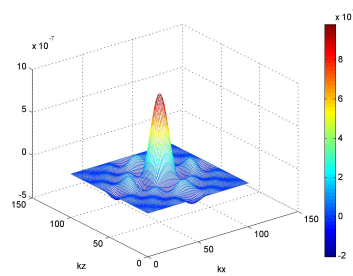


Fig 14: Sinc profile at  $k_z = 0$ . The figure does not show the third sinc function, in the z-dimension.

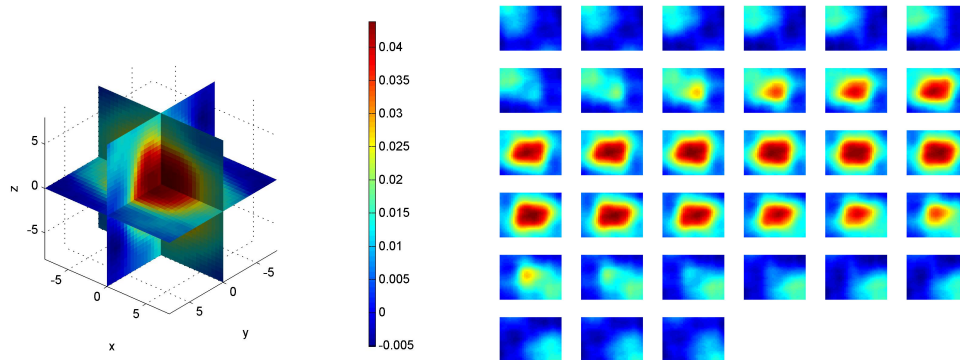


Fig 15: Simulation results for the excitation of a cube.

## 5 Relaxation Effects

The pulse sequence for the excitation of a Gaussian-weighted sphere requires 20ms, under current hardware limitations. This is relatively long as compared to other modern sequences, such as SSFP, and therefore 3D excitation is usually avoided. We tested the robustness of the method for various T2 values, when T1 = 100ms.  $M_y$  magnetization is detectable for up to T2 = 5ms, which is sufficient for most imaging purposes.



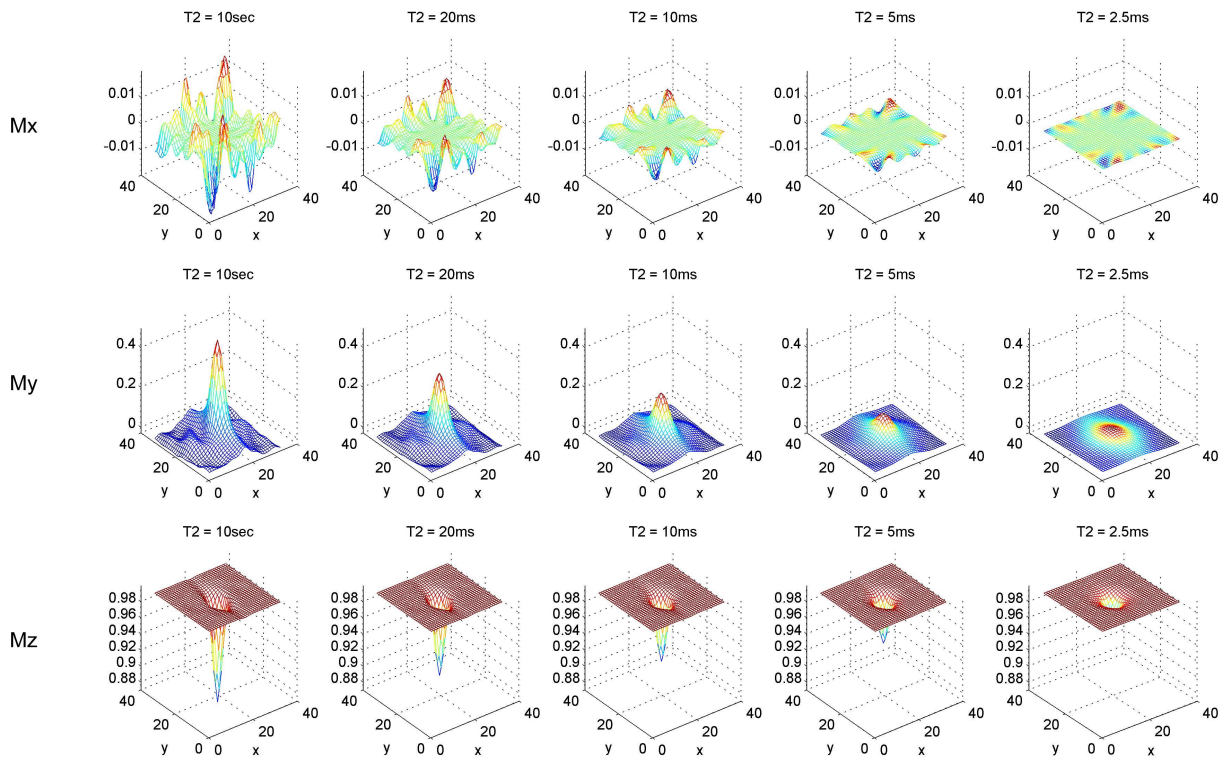


Fig 16:  $M_x$ ,  $M_y$ , and  $M_z$  magnetization under Gaussian-weighted sphere excitation, for various values of  $T_2$ .  $T_1$  was 100ms.

## 6 Of resonance Effects

Unfortunately, due to the long excitation period, the pulse sequence is very sensitive to of resonance. Even 0.25ppm on a 1.5T mashine (1ppm corresponds to  $df = 63.86\text{Hz}$ ) can cause significant artifacts.

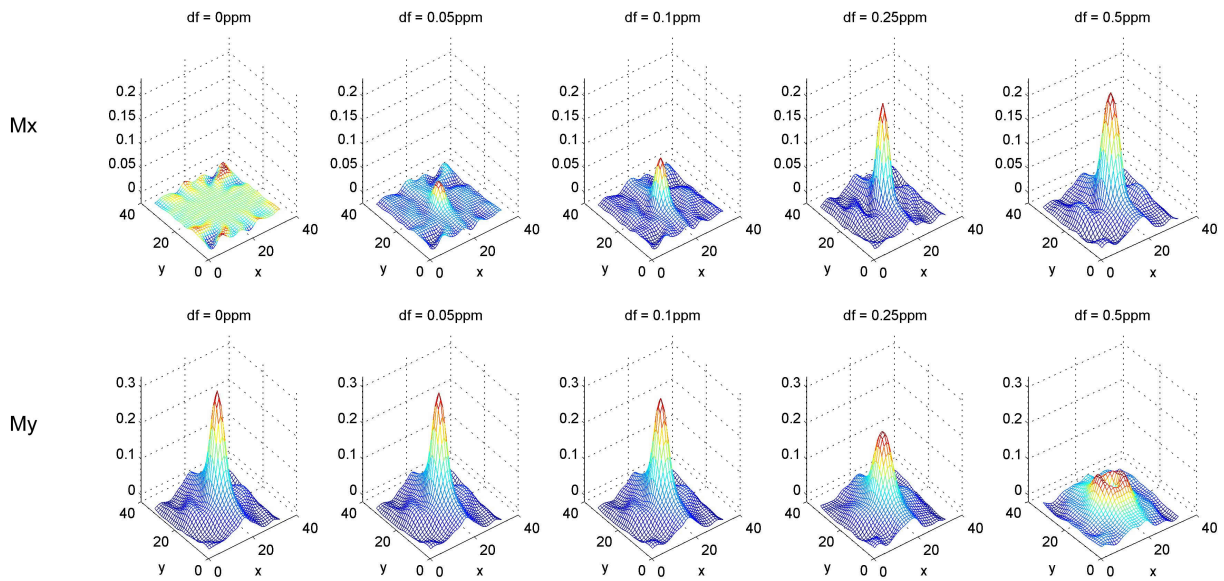


Fig 17: Of resonance behavior of the Gaussian-weighted sphere excitation scheme.

## References

- Bornert, P., Aldefeld, B., December 1998. On spatially selective rf excitation and its analogy with spiral mr image acquisition. *Magma* 7 (3), 166–178.
- Morrell, G., Macovski, A., March 1997. Three-dimensional spectral-spatial excitation. *Magn Reson Med* 37 (3), 378–86.
- Pauly, J., Nishimura, D., Macovski, A., 1987. A k-space analysis of small-tip-angle excitation. *Journal of Magnetic Resonance* 81, 43–56.