EE591 Magnetic Resonance Imaging Term Project Report

Sanghee Cho
sanghcho@usc.edu

Spatially Selective Excitation Pulse Design Based On the k-space Analysis

Abstract

In this project, first of all, a k-space analysis method of John Pauly[1] can be well understood. With this knowledge, several spatially selective pules are designed. To design these, several hardware limitations, maximum gradient level and slew rate, are considered. A systematic approach is also derived to make selective excitation to the desired spatial position and shape. It seems that this design has a reasonable excitation time so could be used a initial estimate for more optimization. While doing this, I had to be very careful for the unit of all parameters.
By designing two types of EPI spatial selective excitation pulses, we also saw the off-resonance effect to each pulse And also, theoretical analysis for the spectral-spatial selective pulse was explained.

I. Introduction

Under the small-tip-angle approximation assumption, we can analyze the selective excitation with the fourier transform relationship between the k-space domain and selected spatial domain. Even, in the class, we have learned that the fourier transform of B1 field shape will decide the slice profile through the z-axis when we excite the thin slice which is perpendicular to the z-axis. In the homework, we had also covered spectral-spatial pulses which can selected not only the thin slice to the z-axis but also spectral band which is helpful to suppress some species in the particular off-resonance.
While doing these two cases, it was very exciting to me to analyze it in the k-space domain with B1 field weighting. But, it was not fully understandable at that time so I chose this study as my term project topic and now, it is clear to me mostly thankful to the John Pauly's paper[1].
In this project, I will show the designed gradients fields and B1 pulse to select the 2D area which is circular shape or rectangle shape with several k-space trajectories. Here, I will also consider the maximum gradients and slew rate which are allowable recently with sum margin. And then, we will see how the off resonance effect affects the excited area. For Echo-Planar 2D-Selective RF Excitation, I will show two pulse shapes which can constitute the trade-off between the time efficiency and the off-resonance. And, I will finish by interpreting the spectral-spatial selective pulses with this analysis method with my understanding.

In section II, I will briefly review the John Pauly's paper[1]. Pencil-beam shape and Rectangle shape excitation pulse design simulation will follow in section III and Off-resonance effect will be examined for those excitations in section IV and spectral-spatial pulse which was in our homework will be analyzed with this analysis method.

II. A k-space analysis of small-tip-angle excitation[1]

When we use the small tip angle approximation, we can write down the signal equation like followng,

$$M_{xy}(\boldsymbol{x})=i\gamma M_0\int_0^T B_1(t)e^{-i\gamma \boldsymbol{x}\cdot\int_t^T G(s)ds}dt \quad (1)$$

,where we assumed that the system is initially in the state (0,0,M0).
We can define the spatial frequency variable k(t) as

$$\boldsymbol{k}(t)=-\gamma\int_t^T \boldsymbol{G}(s)ds \quad (2)$$

Then, we can re-write the equation (1) using delta function like following,

$$M_{xy}(\boldsymbol{x})=i\gamma M_0\int_{\boldsymbol{k}}\{\int_0^T B_1(t)\delta(\boldsymbol{k}(t)-\boldsymbol{k})dt\}e^{-i\boldsymbol{x}\cdot\boldsymbol{k}}d\boldsymbol{k} \quad (3)$$

From eq.(3), we can already know that the signal is the fourier transform of the weighted k-space trajectory by B1(t).
Using the fact, $|\dot{\boldsymbol{k}}(t)|=|\gamma\boldsymbol{G}(t)|$, we can define two function W(k(t)) and S(k) like following,

$$W(\boldsymbol{k}(t))=\frac{B_1(t)}{|\gamma\boldsymbol{G}(t)|}, S(\boldsymbol{k})=\int_0^T\{\delta(\boldsymbol{k}(t)-\boldsymbol{k})|\dot{\boldsymbol{k}}(t)|\}dt \quad (4)$$

And then , finally , we get following nice equation (5).

$$M_{xy}(\boldsymbol{x})=i\gamma M_0\int_{\boldsymbol{k}}W(\boldsymbol{k})S(\boldsymbol{k})e^{-i\boldsymbol{x}\cdot\boldsymbol{k}}d\boldsymbol{k} \quad (5)$$

That is, the transverse magnetization is the fourier transform of a spatial frequency weighting function W(k) multiplied by a spatial frequency sampling function S(k). One thing we have to note from eq.(2) is that the k-space always ends in the origin so we have to proper refocusing to deposit the selected area to the desired position. From eq.(4), weighting function W(k) can be only B1(t) dependent if the gradient is constant when the weighting is deposited. EPI spatial selective pulse and spectral-spatial selective pulse will use this property. And if we well defined the k-space trajectory to cover whole area evenly, we can ignore the S(k) effect. This case is considered in the pencil-beam design to find the B1(t) function. This is pretty simple derivation but the effect is very powerful.

III. Pencil-beam and rectangle shape excitation pulse designed

(a) Pencil-beam Shape

To design correct B1 pulse and Gradients, we have to consider some hardware limitation of the MRI system and also we have our desired shape and size of the selected area. I assumed 3Tesla B0 field and then, the maximum allowed gradient amplitude is 4 G/cm and maximum slew rate is 4G/cm/260usec =15384.62 G/cm/sec. But for slew rate, I used slightly slow value ,that is, 4G/cm/300usec. And also we need to define the FOV and selected area size. 10cm FOV is defined and selected area size is set to a disc of the diameter 3cm. Figure.1 shows this. And one more thing has to be noted is the flip angles which are set $\pi/6$ and $\pi/2$.

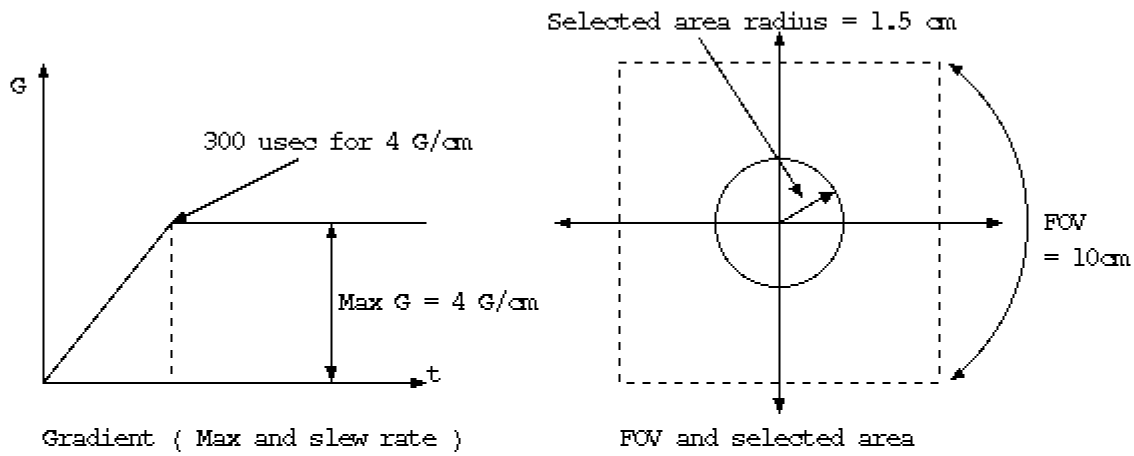


Figure.1 Hardware lemitation and selected area dimension

I will briefly summarize my procedure to design the pulse.

1) Parameter β determination of eq.(20) of [1], which can cover above 95% area with our k-space trajectory.
2) Parameter A selection of eq.(20) which determines the most outter radius of the k-space trajectory, which can be determined by using the inverse fourier transform of the D(k) function (eq.(20 of [1]) and the fact that the diameter of the selected area is 3 cm.
3) With the FOV information, determine n by applying the relationship between the k-space sampling interval and FOV.
4) With A, n, now , we can determine T by setting the gradient peak amplitude to the maximum allowable value which is 2 (considering the negative swing).
5) Then, check the slew_rate of given gradient by approximated equation iteratively by increasing T value step by step until the maximum slew_rate is met.
6) With this value make B1(t) pulse by equation.(21) of [1]. Here, we have to determine the α value for given flip angle, which can be easily found by simple equation.

To design this pulse, first of all, we have to use the well known theory that the fourier transform of the gaussian function is also gaussian itself like below equation (6). (I also write the D(k) function)

$$\boldsymbol{D}(\boldsymbol{k})=\alpha e^{-\beta^2(k_x{}^2+k_y{}^2)/A^2}, F(u,v)=\sqrt{(2\pi)}\sigma e^{-\frac{u^2+v^2}{2\beta^2}} \Leftrightarrow \frac{1}{\sqrt{(2\pi)}\sigma} e^{-\beta^2\frac{(x^2+y^2)}{2}} \quad (6)$$

With D(k) and following integral result for D(k) we can get β to make the k-space trajectory cover above 95% area of this function. It is simple when we use different coordinate system.

$$\int_0^{2\pi}\int_0^{A} \rho e^{-\frac{\rho^2}{(A/\beta)^2}} d\rho d\theta = 2\pi (A/\beta)^2 (1-e^{-\frac{A^2}{(A/\beta)^2}}) \quad (7)$$

So we want the β value satisfying following equation,

$$(1-e^{-\frac{A^2}{(A/\beta)^2}})=0.95 \rightarrow \beta \approx 1.731 \quad (8)$$

In simulation, we used just 2 as the β value for convenience. With similar approach, we can get A value from the inverse fourier transform by using eq.(6) to contain the 95% data deposited in the given circle area.

After setting the A value, we can easily get n value by using following equation,

$$\Delta k \approx \frac{A}{2\pi n} < \frac{1}{FOV} \quad (9)$$

It is simple approximation which we only consider the kx axis (when ky=0) or ky axis (when kx=0) to find the Δk . Next, we have to find the T value. Fortunately, we have equation for it. The maximum peak value of a gradient should be less than 2 G/cm so we can find T value with following equation (10).

$$\frac{A}{\gamma T} \times 2\pi n = 2 \quad (10)$$

Now, we have to check this value with slew rate constraint. It was highly complex to find the maximum gradient from the given gradient function so I used simple approximation like following equation (11).

$$\frac{2 \times Maxx\ Amplitude\ of\ Gradient}{T/(2n)} \leqslant Max.\ slew\ rate \quad (11)$$

So, we iterate by increasing T value step by step until this condition is satisfied. Finally, we determines α value of eqation(1) with given flip angle θ by using following equation.

$$\theta = \gamma \int_0^T B_1(\tau) d\tau \quad (12)$$

With this design method, finally, we get all parameters like table.1. To us, it seems that the total excitation time T is reasonable. So if we want to do more tunning, our method can be used as an initial estimate because our design step is very straight forward so even can be welcome.

Parameters	β	A	n	T	α
Value	2 (rad)	4.616(rad)	8	4.64 msec	6.8276e-07 $(\theta=\pi/6)$
					2.0483e-06 $(\theta=\pi/2)$

Table.1 Final Parameter values

Figure.2 and Figure.3 shows the final result of the transverse magnetization signal. We can see that even the small-tip-approximation is also well applied for the large tip $(\theta=\pi/2)$ case.

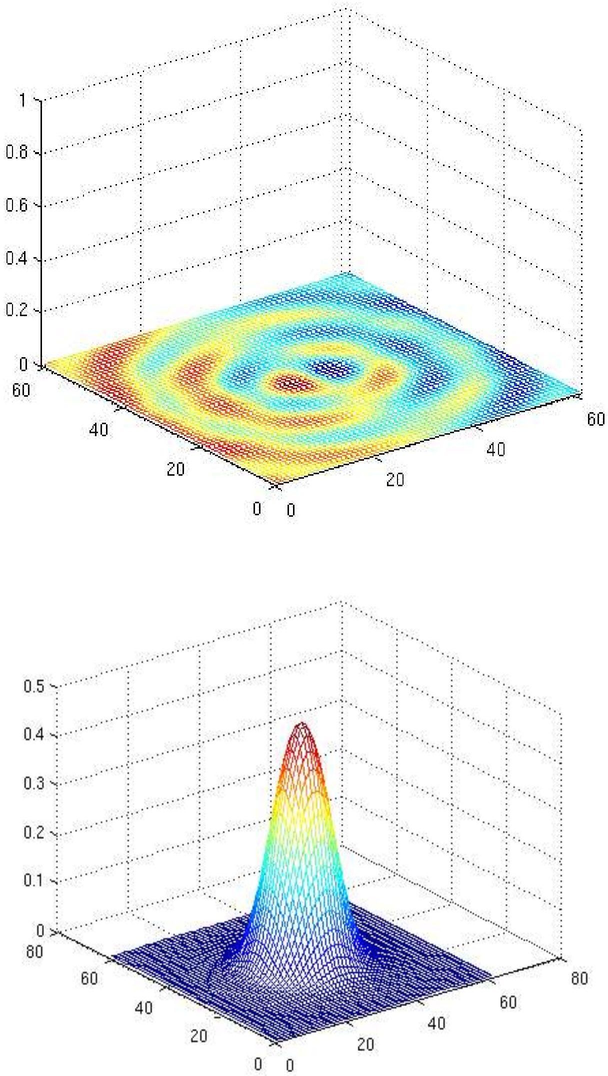


Figure.2 Transverse magnetization for tip angle 30 degree (Mx:above, My:below)

In the figure.2 and 3, the FOV is 6 cm for both axises so that we can well verify that the excited area is about 3cm which is our original objective.

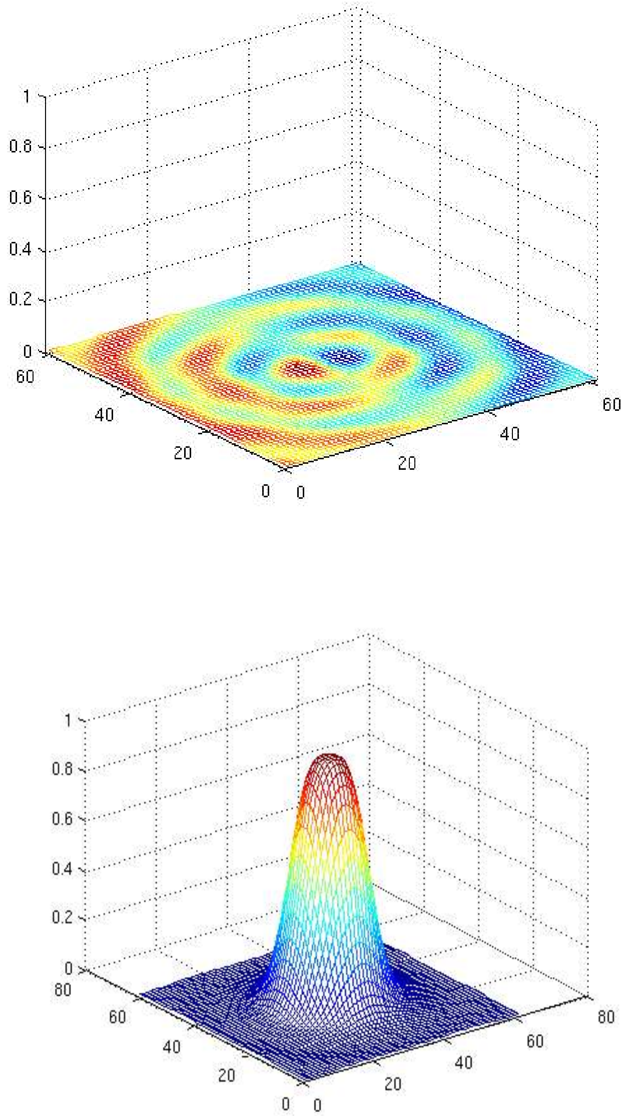


Figure.3 Transverse magnetization for tip angle 90 degree (Mx:above, My:below)

By observing the peak of My signal, we can verify the tip angle for each case, too.

(b) Rectangle Shape

As we may know, the covered area by k-space trajectory in the EPI is the rectangular shape so we have to utilize the B1 pulse to weight the k-space trajectory properly to make rectangle area selected in the spatial domain. With these weightings for both directions, we can easily infer how and which area will be excited selectively in the spatial domain if we take the fourier transform for each signal. Intuitively, we will get replicated pattern

through the phase encoding direction because it is a sampled function. Figure.4 explains this.
Our objective is to selectively excite the spatial rectangle area of Fx = 1 cmd and FOVy=10 cm (Fy=5cm, with our given phase encoding number). We will design two types of pulses which are shown figure.5 and 6. The first type has a robustness for gradient imperfections because most of them affect k-space traverses of equal directions in nearly the same way. The second pulse type in Figure.5 is more time efficient but it can be more sensitive to the off-resonance effect.[3]

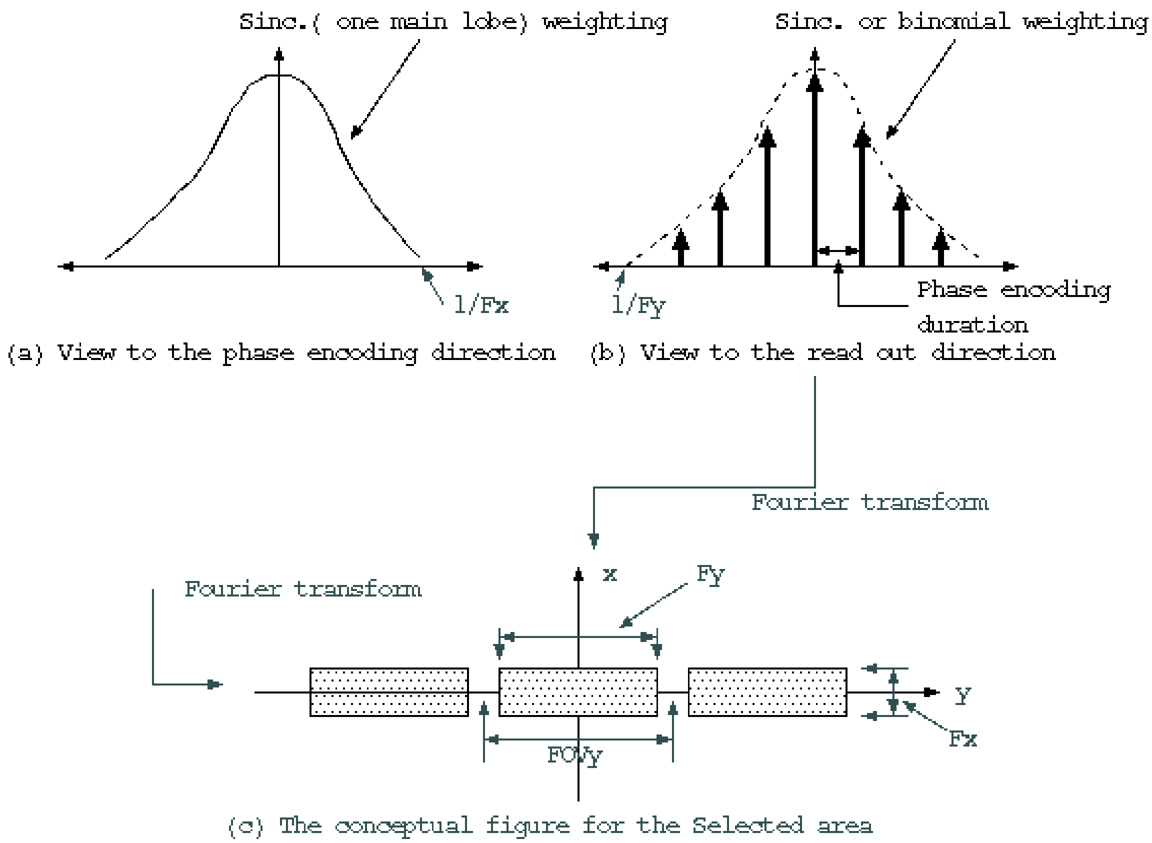


Figure.4 B1 pulse weightings ((a) and (b)) in the k-space trajectory for EPI and rough structure of selected area ((c)) in the spatial domain

However, these two cases are not much different in the design so I will first explain with the second pulse type case and also with keeping in mind the hardware limitation which are same with pencil-beam case. The brief summary for the design will be following.

1) Gx Gradient design : It is obvious if we consider the maximum slew rate and Fx. The peak Gradient Gxm will be 2 and the duration for plateau of the gradient will be calculated by following equation.

$$\gamma G_{xm} T = W_{k_x} = 2 \times \frac{2\pi}{F_x} \quad (13)$$

2) Refocusing Gx Gradient
3) Gy Gradient design : Fix the slope with the maximum slew rate then, adjust the peak Gym to meet the FOVy=10cm. So the triangle pulse duration T will be calculated by following equation. Because we used 6 phase encodings so the half width of the envelope of the weighting is $4\Delta k_y$, therefore Fy = 2/($4\Delta k_y$) =FOVy/2=5.0cm. This is, once we determine FOVy and the total number of phase encodings, we can find the Fy. Of course, we can find FOVy if we already determined the Fy and the total number of phase encodings.

$$\gamma T G_{ym} = 2\pi \Delta k_y = \frac{2\pi}{FOV_y}, \frac{G_{ym}}{T} = slew\, rate \quad (14)$$

4) Refocusing Gy Gradient
5) B1 RF pulse design : Used one main lobe of a sinc pulse whose width is same with the pateau of the Gx gradient. And we need to weight these sinc pulses with sinc, gaussian or binomial coefficients to get correct shape through the phase encoding direction. In our case, we used sinc. function weighting. Of course, there should be a scaling for B1(t) to meet the flip angle.

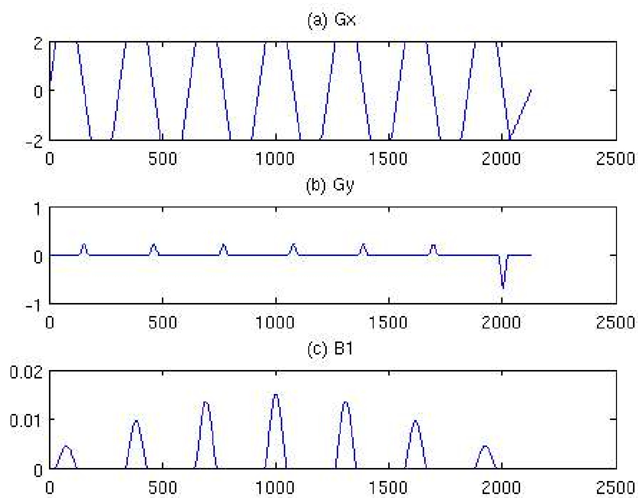


Figure.4 EPI spatially selective pulse (I)

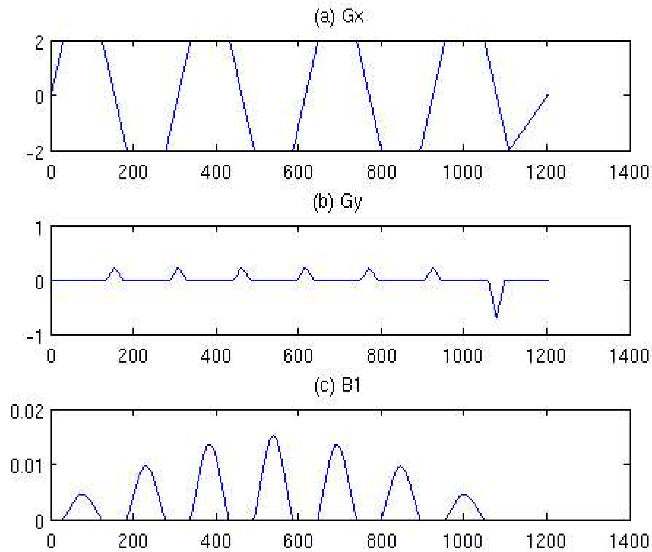


Figure.5 EPI spatially selective pulse (II)

We used the total 7 RF pulses so the total duration for this excitation is (150+470+150) x7+(150+470) = 6010 usec for pulse type II and (150+470+150)x13+(150+470)=10630 usec for pulse type I.

When there is no off-resonance the resulting transverse magnetization signals are same. Figure.6, 7 shows the resulting signal for different flip angles (30 and 90 degree). FOVy is 10 cm and the figure shows the data in the scope of 20 cm so clearly we can see that replicated pulses due to the sampling through the phase encoding direction.

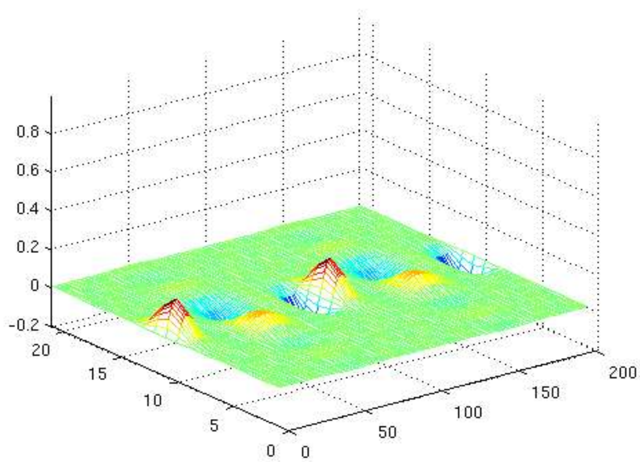
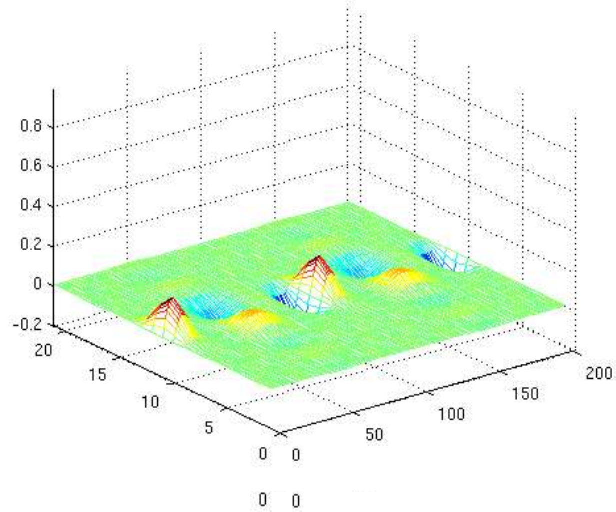


Figure.6 Transverse magnetization for tip angle 30 degree (Mx:above, My:below)



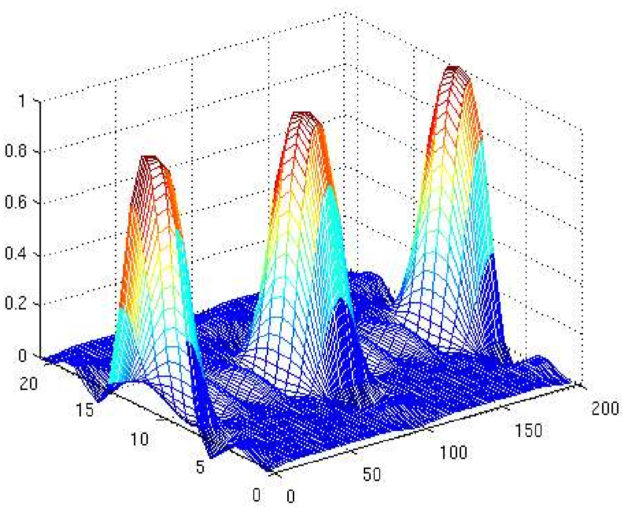


Figure.7 Transverse magnetization for tip angle 90 degree (Mx:above, My:below)

For the read-out direction, the total scope is 2cm so we can verify that the 1cm Fx was excited selectively. And also Fy=5cm is verified. Here, also the small-tip-angle approximation analysis is well applied even to the 90 degree.

IV. Off Resonance Effect for each pulse

To the spiral trajectory excitation, 25hz off-resonance was applied with 90 degree flip angle. Figure.8 shows the signal Mx and My. Mx signal is quit noticeable which is not non zero for selected area. It is not easy to analyze this result, but off-resonance causes the precession along the z-axis , which affects the signal amplify for the Mx signal.

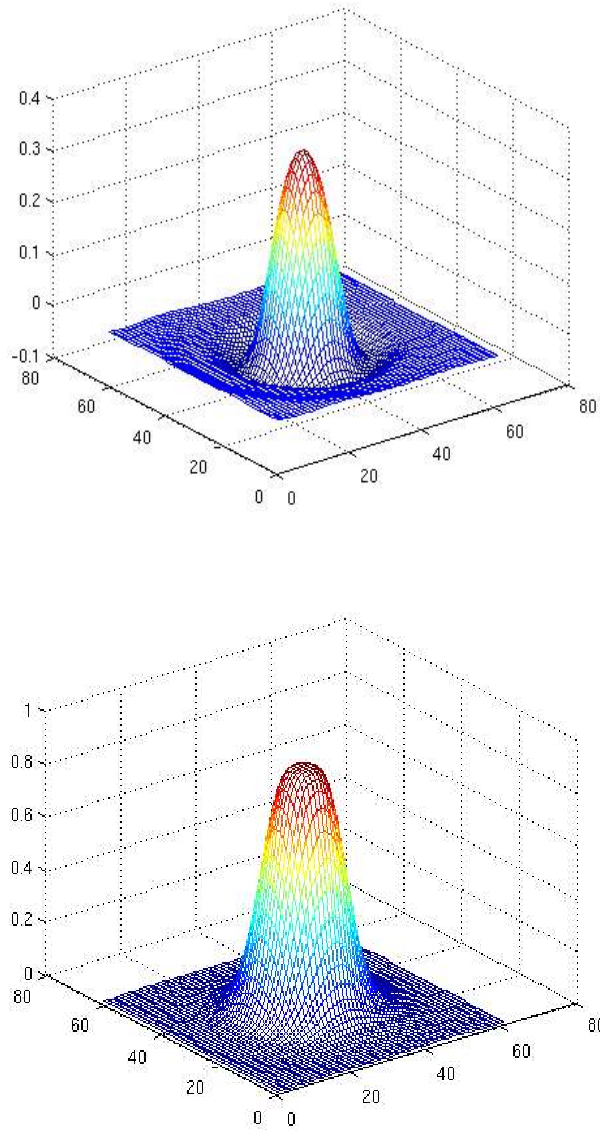


Figure.8 Transverse magnetization of pencilbeam excitation pulse for 25Hz off-resonance (tip angle 90 degree)(Mx:above, My:below)

For the EPI spatial selective pules, Figure.9 and 10 show the signal with 25hz off-resonance for type I and type II pulse. I anticipated type II pulse will show poor signal because it has more time efficiency than type I but I couldn't find particular difference between two. There might be some problem with my code or knowledge. However, it is interesting that the Mx and My signal has same shape and even both signals has a shifted form from the non off-resonance signal to the phase encoding direction.

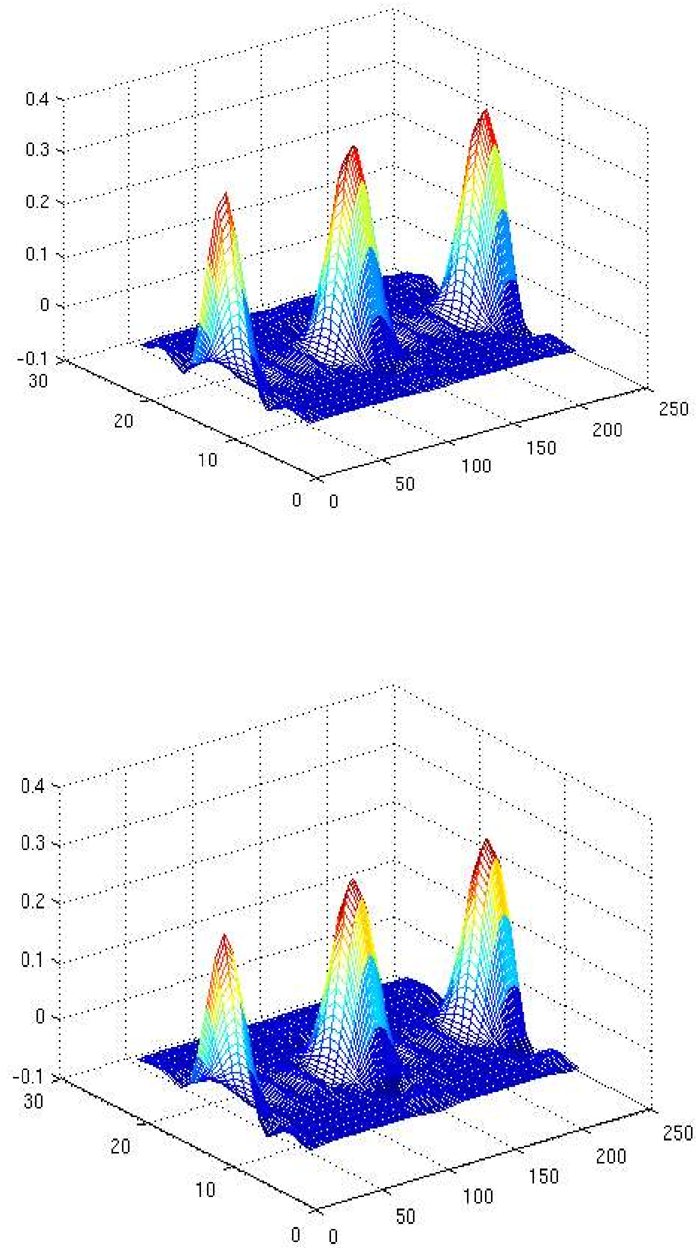
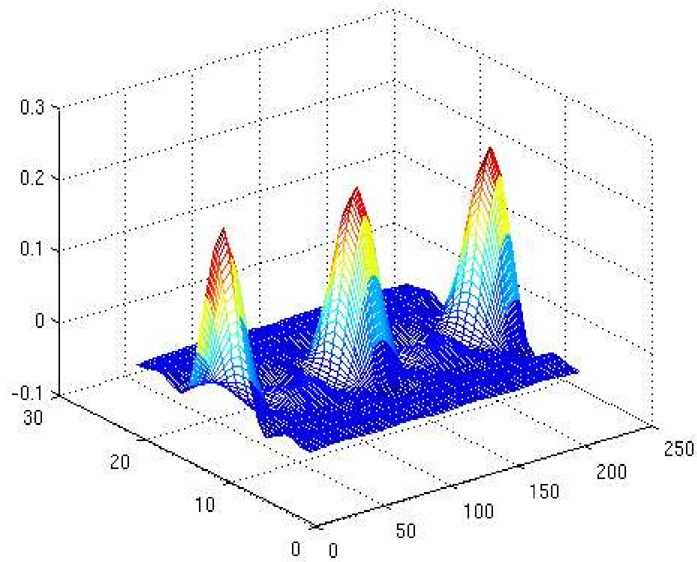


Figure.8 Transverse magnetization of type I pulse for 25Hz off-resonance (Mx:above, My:below)



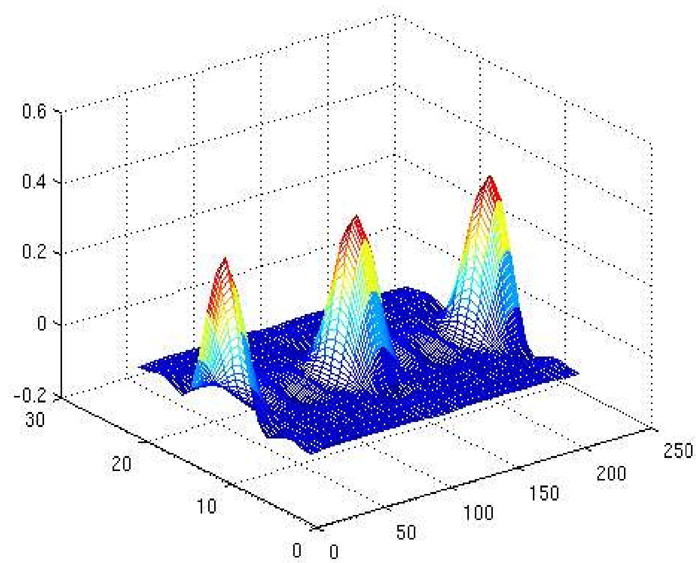


Figure.9 Transverse magnetization of type II pulse for 25Hz off-resonance (Mx:above, My:below)

V.Spectral-Spatial Pulse Interpretation with k-space analysis

In the homework, we had a pulse which is shown in the figure.10, but this is very simlar to the EPI spatial selective pulse. As an alternative, I copied another 'specspat2.mat' file from our course website which is a little bit different from the previous one. Figure.11 shows this pulse shape.

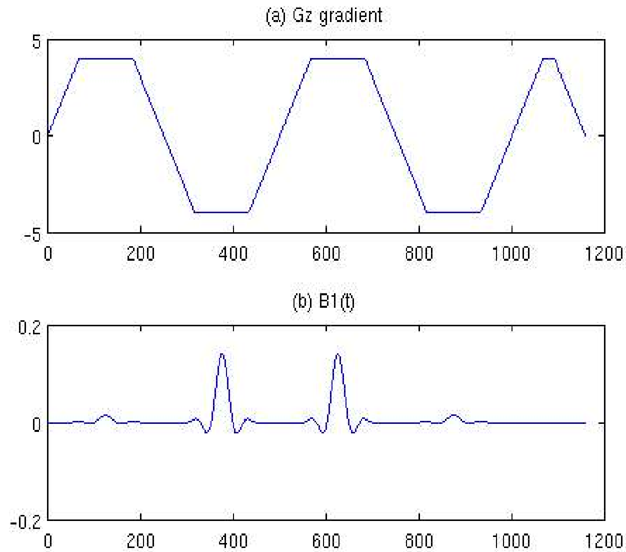


Figure.10 Specspat.mat pulse in our course website

For figure.7 case, we have to consider Gz field for weighting function W(k) of eq.(4). But we can still regard this weighting function as a kind of gaussian function. So we can take same approach to analyze it, which is taken for EPI spatial selective pulse in figure.4 except changing the phase encoding direction with time direction.

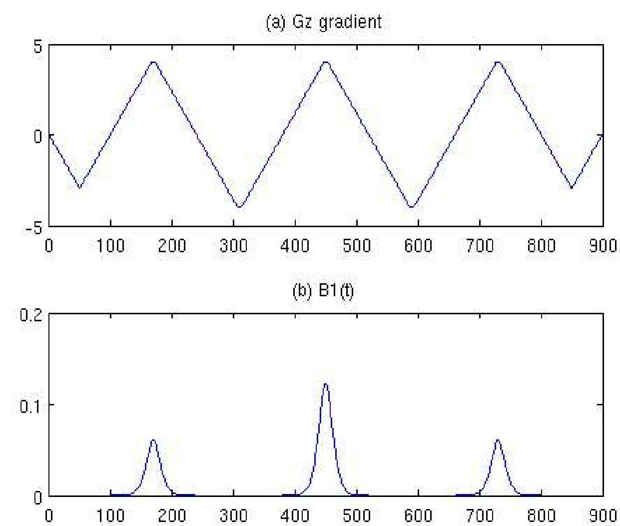


Figure.11 Specspat2.mat pulse in our course website

I summarized the analysis in the figure.9. As the this figure shows, we can know that the cosine type function will be generated through the frequency axis which will pass or suppress some frequency bands.

If we also apply the fourier transform characteristics, we can adjust the suppressed frequency by applying the linear phase to the time direction weighting, which causes the shift in the frequency domain.

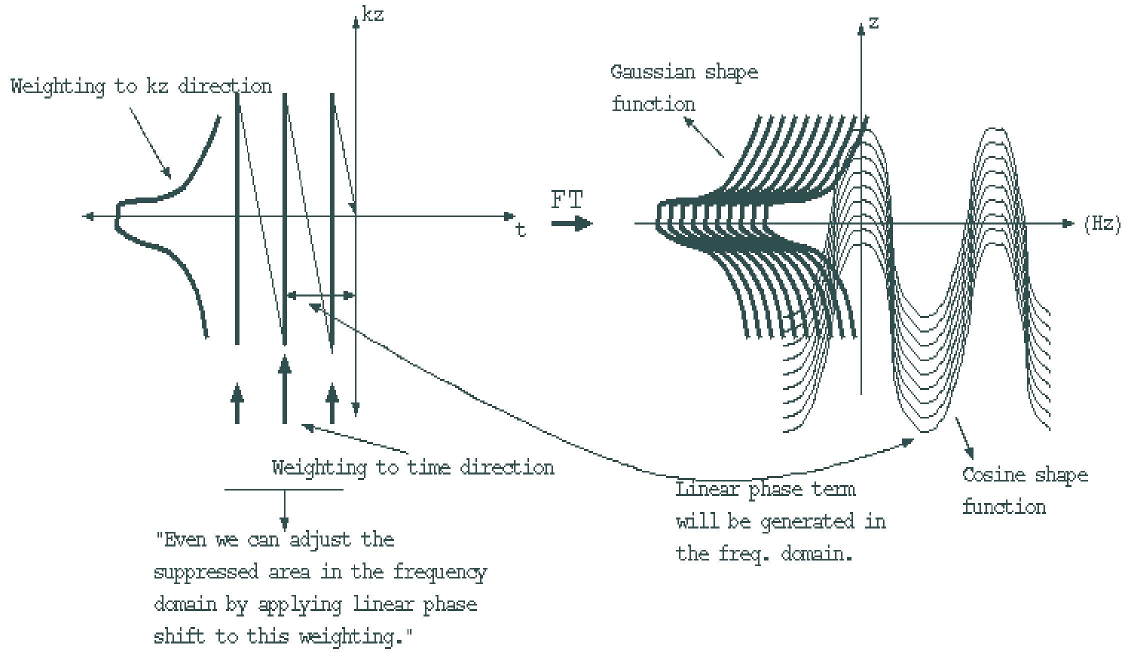


Figure.12 Specspat2.mat pulse analysis

VI. Conclusion

John Pauly's k-space analysis was studied. Even though it has small-tip-angle assumption, it is also well applied to the large tip angle(in our simulation, up to 90 degree). This idea is quit simple but enough as a design tool at least at the first stage to give an rough insight to us.

Several excitation pulses were simulated and their off-resonance effects were mentioned but it is somewhat insufficient. By time restriction, I couldn't give full analysis for the off-resonance effect which could be more important one. However, I got full acknowledge about this analysis and also added the analysis in my word for spectral-spatial pulse which is in our course website.

And I derived a systematic way to design each pulse with given spec. (the area size which will be selected or FOV etc.) as well as the hardware limitations (Maximum gradient limitation and maximum allowable slew rate).

References

[1] Pauly J, Nishimura D, Macovski A. A k-space analysis of small-tip-angle excitation. J Magn Reson 1989;81:43-56

[2] Rieseberg S, Frahm J, Finsterbusch J. Two-Dimensional Spatially-Selective RF Excitation Pulses in Echo-Planar Imaging. Magn Reson in Med 2002;47:1186-1193

[3] Oelhafen M, Pruessmann K, Kozerke S, Boesiger P. Calibration of Echo-Planar 2D-Selective RF Excitation Pulses. Magn Reson in Med 2004;52:1136-1145

[4] Grieve S, Blamire A., Styles P. Elimination of Nyquist Ghosting Caused by Read-Out to Phase-Encode Gradient Cross-Terms in EPI. Magn Reson in Med 2002,47:337-343

[5] Zur Y. Design of Improved Spectral-Spatial Pulses for Routine Clinical Use. Magn Reson in Med 2000; 43:410-420