Application of UNFOLD to Spiral Sampling Trajectories

Taehoon Shin

Introduction

In real-time MR imaging applications such as cardiac imaging, the reduction of scan time is an important issue because temporal variation should be resolved as finely as possible. For that purpose, therefore, many different approaches have been reported, including partial k-space reconstruction (1), variable density sampling(2), parallel imaging such as SENSE and SMASH(3), and UNFOLD(4, 5). Among them, UNFOLD is attractive because it can accelerate scan time using purely signal processing technique which exploit efficient use of k-t space.

One of simple usages of UNFOLD is to halve the scan time in 2DFT cardiac imaging as proposed in (1). However there already exist more efficient sampling trajectories like spirals and therefore the use of UNFOLD in 2DFT cardiac imaging might not be satisfactory enough. Then the possibility of applying UNFOLD to spiral sampling trajectories deserves special notice so that the profit from this novel method could be maximized.

Theory

In order to rigorously verify the availability of UNFOLD in spiral sampling trajectories, it is necessary to analyze the effect of spiral sampling in k space on the image domain. In fact the analytic solution of closed form, e.g. the complete formula describing Fourier transform of spirals, was reported from the study on the diffraction pattern in radar application(6). However it doesn't seem to be practically useful due to the excessive complexity of that formula even under the simple condition that the number of spirals is only two. As an alternative model, therefore, concentric ring samples deserve notice since its distribution pattern is similar to that of spirals and relatively easy to handle.

Effect of shifted ring sampling

Because of its property of circular symmetry, the Fourier transform in polar coordinate, called Hankel transform should be used for the study on the effect of ring sampling on the space domain. If the ring distribution for a radial spacing Δ_r is represented by

$$S(k_r) = \sum_{n=1}^{M} \delta\left(k_r - n\Delta_r\right) \quad \text{where } M \text{ is the number of concentric rings} \qquad [1]$$

its Hankel transform, e.g. point spread function(PSF) can be easily found from the basic transform pair $\delta(k_r - a) \leftrightarrow 2\pi a J_0(2\pi a r)$ (7) as follow

$$PSF(r) = F^{-1}\{S(k_r)\} = 2\pi\Delta_r \sum_{m=1}^{M} nJ_0(2\pi rm\Delta_r) \qquad [2]$$

where J_0 is Bessel function of the 1st kind of order 0

For the number of rings M=15 and radial spacing Δ_r=0.1, the PSF profile expressed above is displayed in Fig. 1. As expected there exist a big peak at the center (main lobe) and smaller sidelobes periodically in the radial direction.

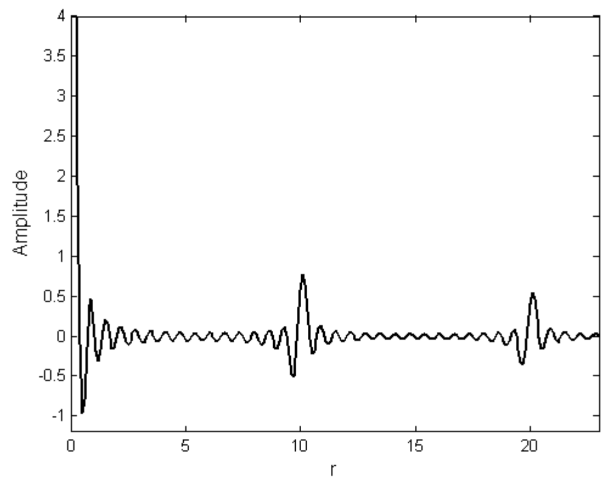


Figure 1. PSF profile of ring trajectories

My interest is actually what would happen when the series of rings is shifted by half the radial spacing in the radial direction. In this case the summation of Bessel functions should be modified as follow

$$PSF_{shift}(r) = 2\pi\Delta_r \sum_{m=1}^{M} (m-0.5)J_0(2\pi r(m-0.5)\Delta_r) \qquad [3]$$

Fig. 2 represents the PSF profile from this shifted ring samples together with the original one where M is reduced into 10 just for simpler shapes.

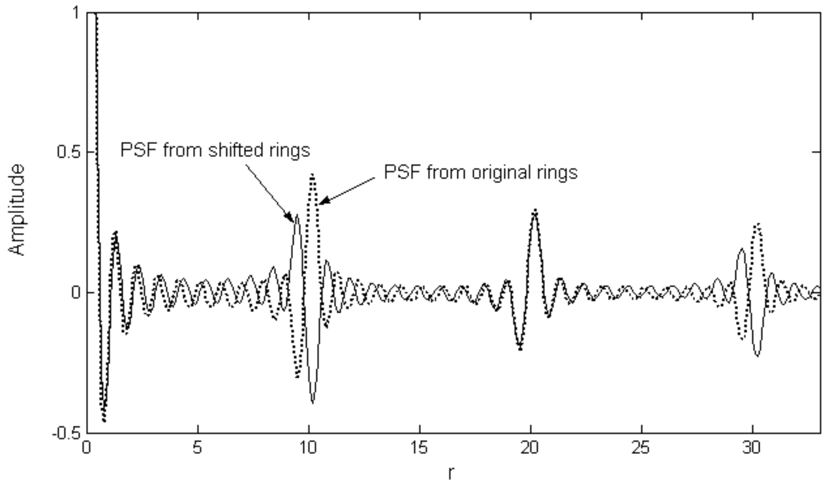


Figure 2. PSF profile of shifted ring trajectories with original one

Interestingly the phase difference is exactly π around the first sidelobe while two signals are in phase around the main lobe. Therefore when the shifted samples are used together with original ones alternatively the aliased image points will be modulated to half the Nyquist frequency, which guarantees the effectiveness of UNFOLD scheme. In fact the sidelobes keep occurring in the radial direction with smaller and smaller magnitudes and two profiles are in and out of phase at the even and odd lobes respectively.

Analysis of the behavior at mainlobe and sidelobes

The desired property in the PSF of the shifted ring samples is already shown through numerical computation of Bessel functions and its resulting plot in the previous section. Here I will derive the analytic formulation of the behavior at mainlobe and sidelobes of both $PSF(r)$ and $PSF_{shift}(r)$.

In general the asymptotic approximation for the Bessel function of k[th] order is represented as (8),

$$J_k(t) \approx \frac{1}{\Gamma(k+1)}\left(\frac{t}{2}\right)^k \quad \text{for } t<<1 \text{ and}$$

$$J_k(t) \approx \sqrt{\frac{2}{\pi x}}\cos\left(t-\frac{k\pi}{2}-\frac{\pi}{4}\right) \quad \text{for } t>>1 \qquad [4]$$

The behavior at mainlobe, e.g. the same magnitude in both PSFs can be easily seen since, for zeroth order (k=1),

$$PSF(r) = PSF_{shift}(r) \approx 2\pi\Delta_r M\frac{1}{\Gamma(1)} = const \quad \text{near the origin(r<<1)} \qquad [5]$$

Using the second approximation $PSF(r)$ and $PSF_{shift}(r)$ far from the origin, can be expressed as follow,

$$
\begin{aligned}
PSF(r) &= \sum_{m=1}^{M} m \sqrt{\frac{2}{\pi(2\pi r m \Delta_r)}} \cos\left(2\pi r m \Delta_r - \frac{\pi}{4}\right) \\
&= \frac{1}{\pi}\sum_{m=1}^{M} \sqrt{\frac{m}{r\Delta_r}} \cos\left(2\pi r m \Delta_r - \frac{\pi}{4}\right) \\
&= \frac{1}{\pi}\sum_{m=1}^{M} \sqrt{\frac{m}{x}} \cos\left(2\pi\, m\, x - \frac{\pi}{4}\right)
\end{aligned} \qquad [6]
$$

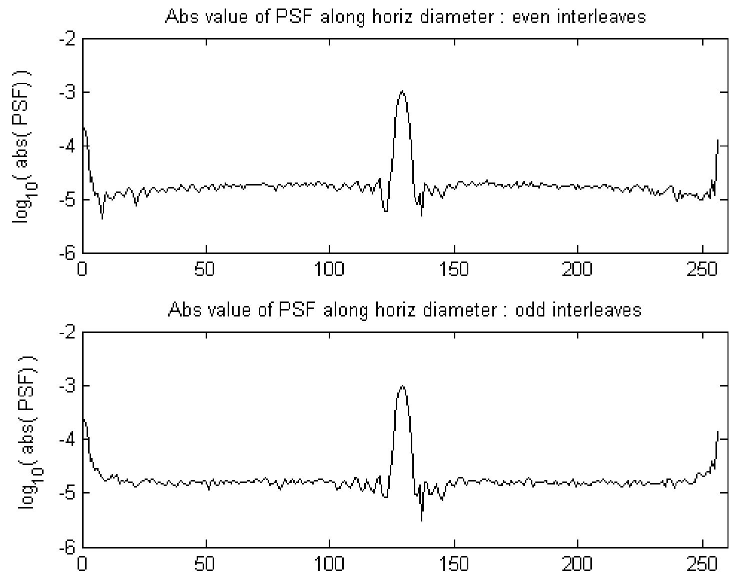
$$
\begin{aligned}
PSF_{shift}(r) &= \sum_{m=1}^{M} m \sqrt{\frac{2}{\pi(2\pi r \Delta_r)(m-0.5)}} \cos\left(2\pi r (m-0.5)\Delta_r - \frac{\pi}{4}\right) \\
&= \frac{1}{\pi}\sum_{m=1}^{M} \sqrt{\frac{m^2}{(m-0.5) r\Delta_r}} \cos\left(2\pi r m \Delta_r - \pi r \Delta_r - \frac{\pi}{4}\right) \\
&= \frac{1}{\pi}\sum_{m=1}^{M} \sqrt{\frac{m^2}{(m-0.5)x}} \cos\left(2\pi\, m\, x - \pi x - \frac{\pi}{4}\right) \\
&= \begin{cases} \frac{1}{\pi}\sum_{m=1}^{M} \sqrt{\frac{m^2}{(m-0.5)x}} \cos\left(2\pi\, m\, x - \frac{\pi}{4}\right) & \text{when } x \text{ is even} \\ \frac{1}{\pi}\sum_{m=1}^{M} \sqrt{\frac{m^2}{(m-0.5)x}} (-1) \cos\left(2\pi\, m\, x - \frac{\pi}{4}\right) & \text{when } x \text{ is odd} \end{cases}
\end{aligned} \qquad [7]
$$

Here $x = r\Delta_r$ is a dimensionless variable such that the PSF shows the peak at its integer values. Comparing [6] and [7] I can see that $PSF(r)$ and $PSF_{shift}(r)$ have almost the same amplitude with the same phase at the sidelobes of even order and opposite phase at the sidelobes of odd order. As expected, therefore, this analytic result is consistent with the salient characteristics at the mainlobe and sidelobes in Fig 2.

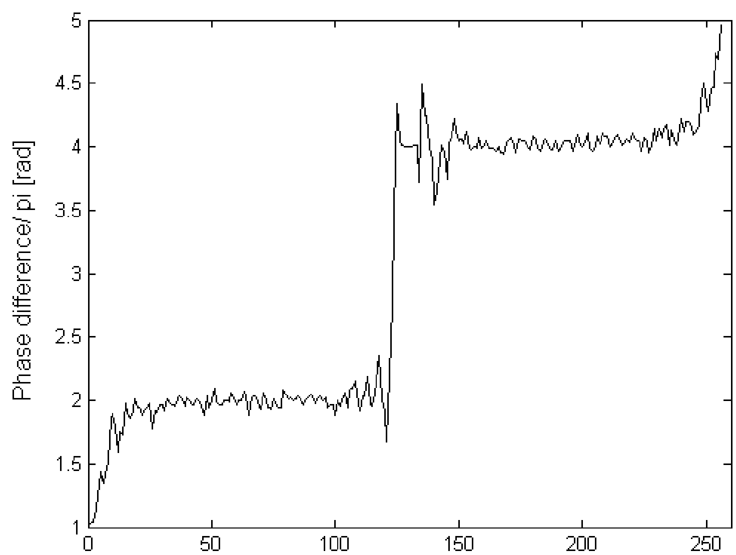
Simulated PSF of spiral trajectories

In the previous section, from both analytic formulation and simulation of PSF, it was proven that UNFOLD algorithm must be effective in concentric ring trajectories. In order to show that this result can carry over to the spiral trajectories, I simulated the PSF of spiral trajectories and examined its magnitude and phase response.

Out of total 12 spiral interleaves provided in the data file 'cor2.mat' six odd numbered interleaves and six even numbered interleaves are used separately for constructing two PSFs. Pre density correction and oversampling are used for gridding reconstruction. Fig. 3 shows the magnitudes of two PSF and their phase difference along horizontal diameter. As expected, the magnitudes are almost same and the phase difference at the sidelobe is exactly π.



(a) Magnitude of each PSF



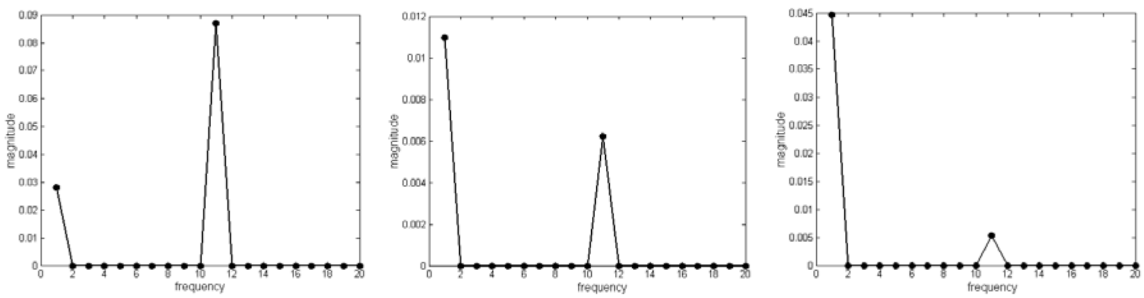
(b) Phase difference between two PSFs

Figure 3. PSFs of even numbered and odd numbered interleaves (along horizontal diameter)

Simulation results

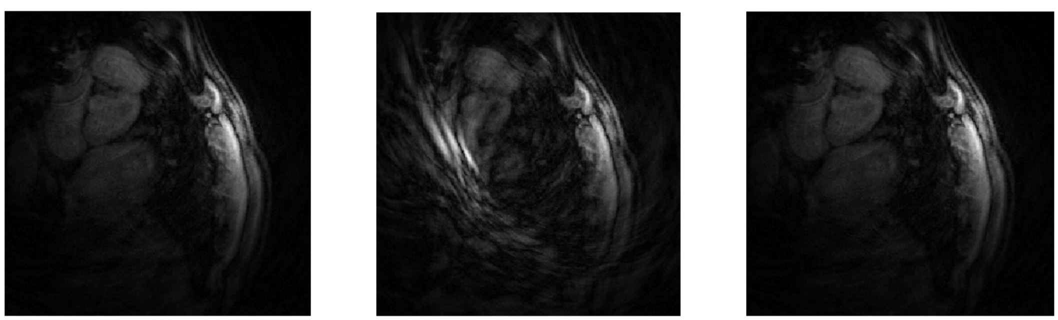
Static image

Basically UNFOLD is the method for dynamic image sequence since it uses the time axis for saving the required space in image domain. Nevertheless the static image can be considered as the simplest example for testing the effectiveness of UNFOLD in spiral imaging. That is, using the single image obtained from the data file 'cor2.mat' I create an image frame consisting of a series of the same still images up to 20 (the number of frames). In the same way as the PSF analysis of the previous section I took the interleaves of even and odd orders alternatively over the time frames and FFTed each image point along time axis. Fig. 4 represents the FFT results at three different image points. Since the constructed image sequence is same over the time axis only DC components of the original and aliased image exist and therefore the perfect reconstruction is possible as shown in Fig. 5.



(a) (64, 64) (b) (64, 5) (c) (5, 64)

Figure 4. Frequency distribution along time axis at different image points



(a) original (b) aliased (half interleaves) (c) reconstructed from UNFOLD

Figure 5. Images at different stage during the application of UNFOLD to static image

Dynamic phantom image sequence

Since the real cardiac image sequence is not available right now, I created phantom image sequence which imitates cardiac motion and breathing. That is, single ring signal is used for modeling an human heart and it shows vertical movement as well as contraction/ expansion both of which are periodic of course. Since the area of the ring signal should be conserved its thickness should be changed dependent on the outer or inner radius. Several image parameters and sequence parameters assumed are summarized in Table 1.

Table 1. Image and sequence parameters

Period of cardiac motion (contraction/ expansion)	0.6 s
Period of breathing (vertical motion)	6 s
FOV	20 cm
Max / min of outer radius	3.91 / 3.28 cm
Range of vertical motion	3 cm
Total number of spirals	6
TR	16 ms

The total number of image frames generated is 30. How the spiral interleaves are collected to form each frames, is represented in Fig. 6 where six numbers 1~ 6 indicate six spiral interleaves. The boxed numbers represent the frames obtained from all six interleaves and they will be called original image frames as reference from now on. The circled numbers denote the frame number for UNFOLD which uses odd and even ordered interleaves alternatively. The number of frames in UNFOLD is twice that of the original frames assuming the same scan time. Of course I can half the scan time if I keep the number of frames of UNFOLD same as that of original frames. Finally I also reconstructed another reference image frames called ground truth images from taking all six interleaves *simutaneously* so that they do not suffer from motion artifact.

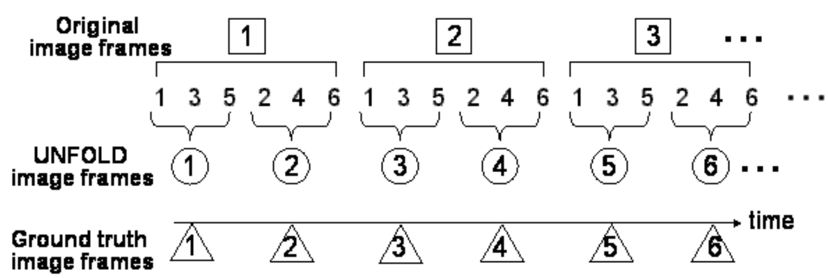


Figure 6. Sequential order of spiral interleaves

Those three different kinds of image frames together with aliased ones obtained from only three interleaves without applying UNFOLD, are shown in Fig. 7 ~ Fig. 10. The true images in Fig. 7 show perfect reconstruction without any motion artifact nor aliasing. The ring signal experiences contraction/ expansion and slow vertical movement as desired. Original image frames in Fig. 8 are free from aliasing but suffer from motion artifact because the six interleaves are sampled at different time. UNFOLD image frames in Fig. 9 are free from aliasing even though only three interleaves are used for each frame and experience slight motion artifact. On the whole the quality of resulting images is comparable to that of the original image frames.

In order to imitate more realistic situation I added static background with the shape of checkerboard and the resulting image frames are shown in Fig. 11 ~ Fig. 13. There must be motion artifact especially around the ring signal in both original and UNFOLD image frames. But they are not clear enough due to overlapping with the background.

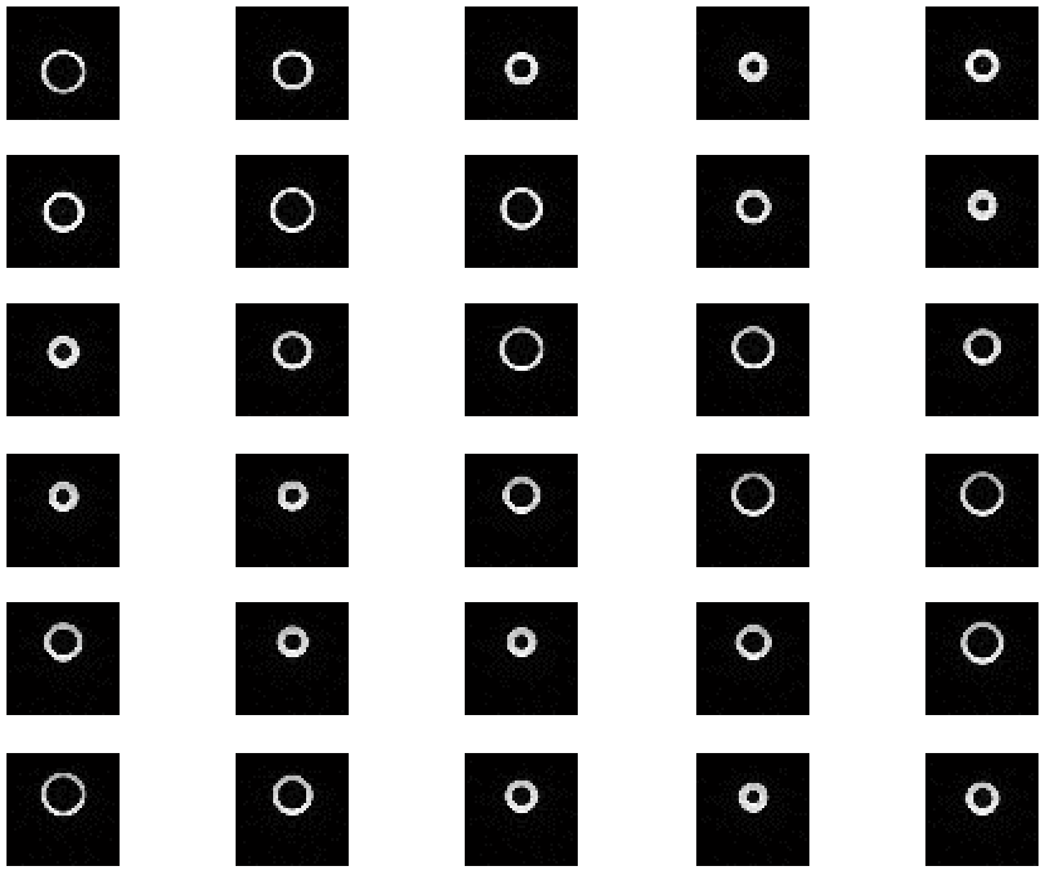


Figure 7. Ground truth image sequence

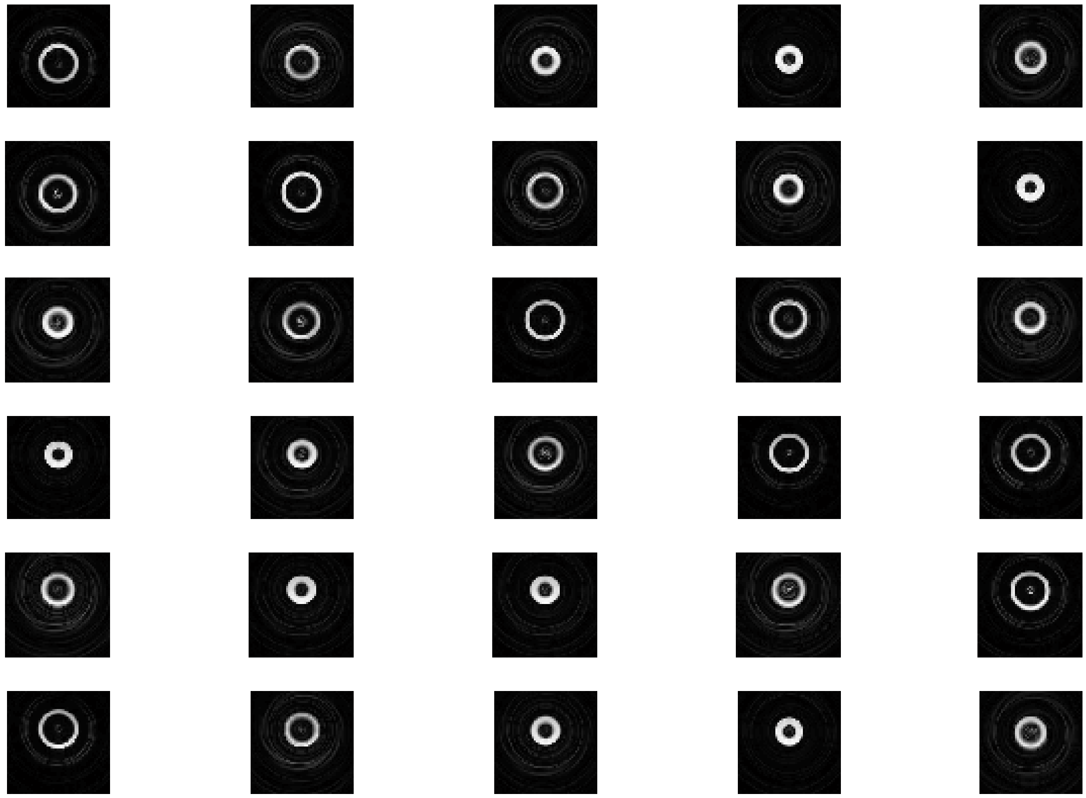


Figure 8. Original image sequence obtained in the conventional way

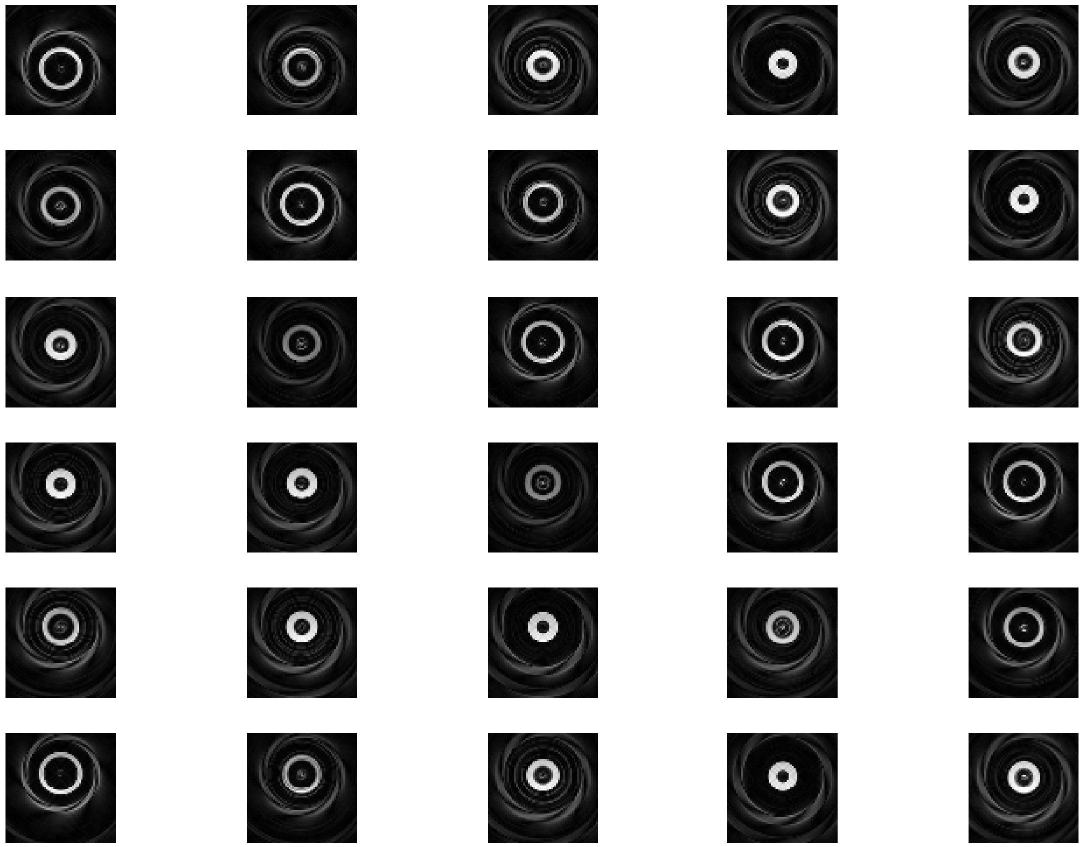


Figure 9. Aliased image sequence obtained from half interleaves only

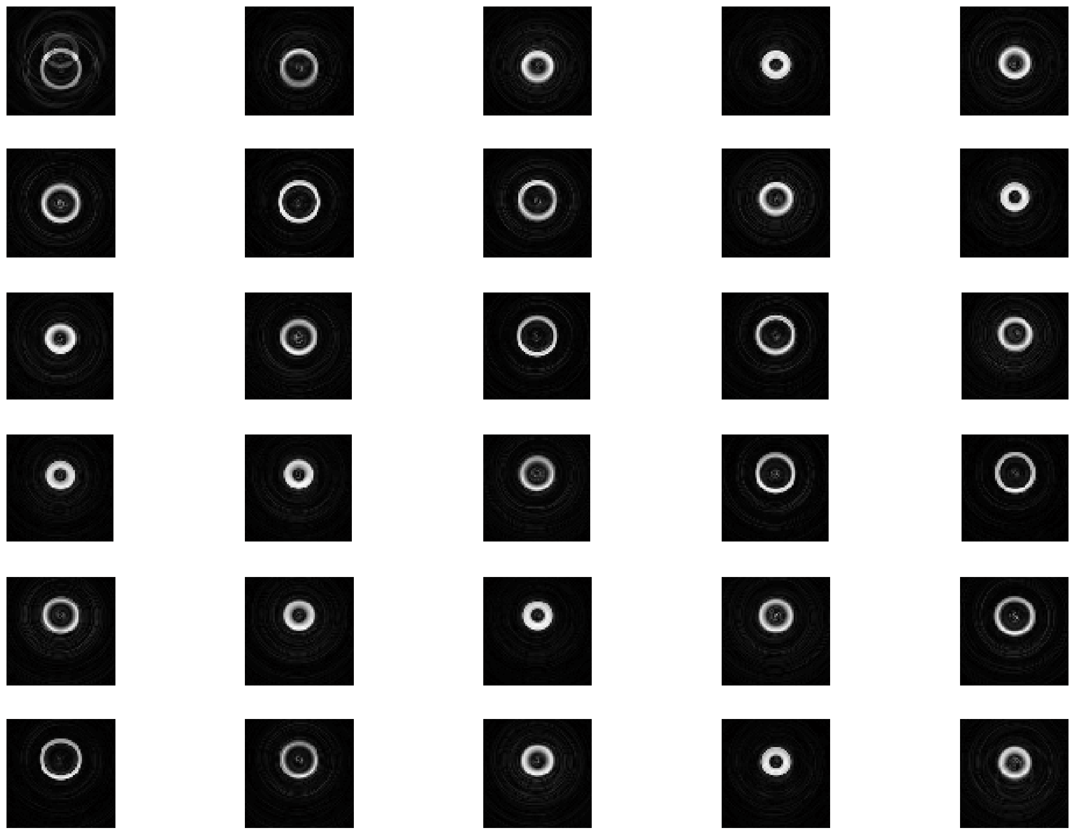


Figure 10. Image sequence obtained from UNFOLD

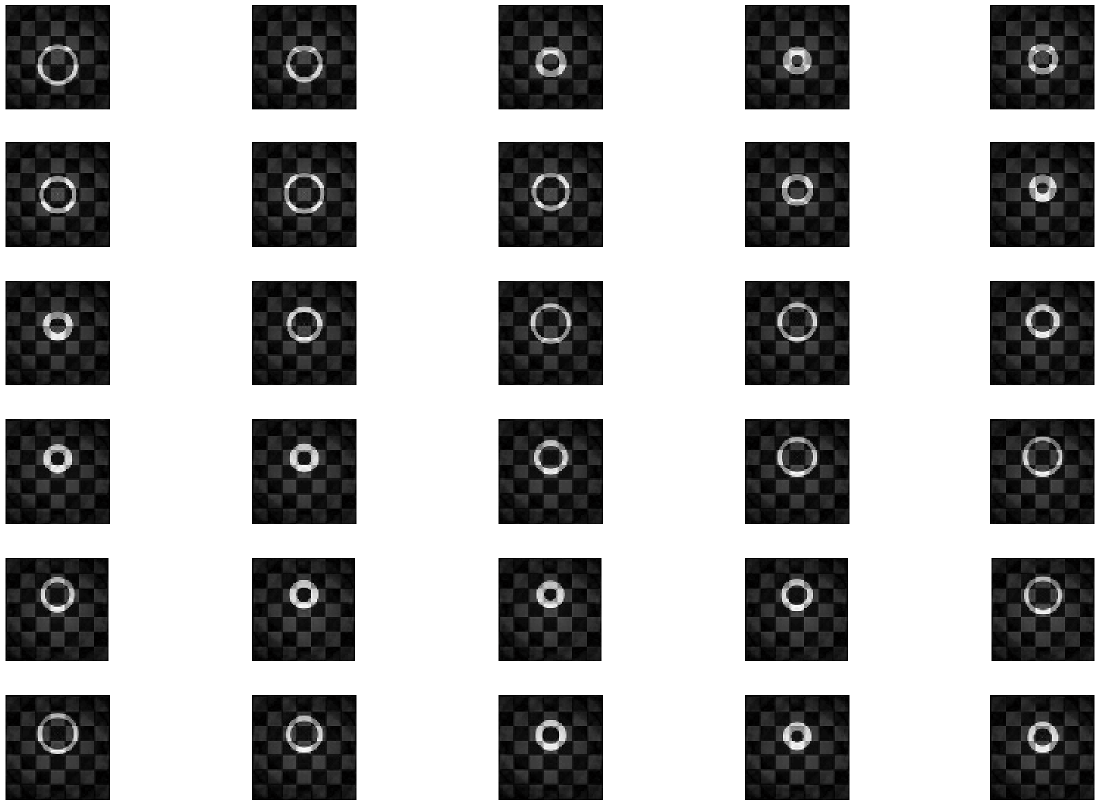


Figure 11. Ground truth image sequence with static background

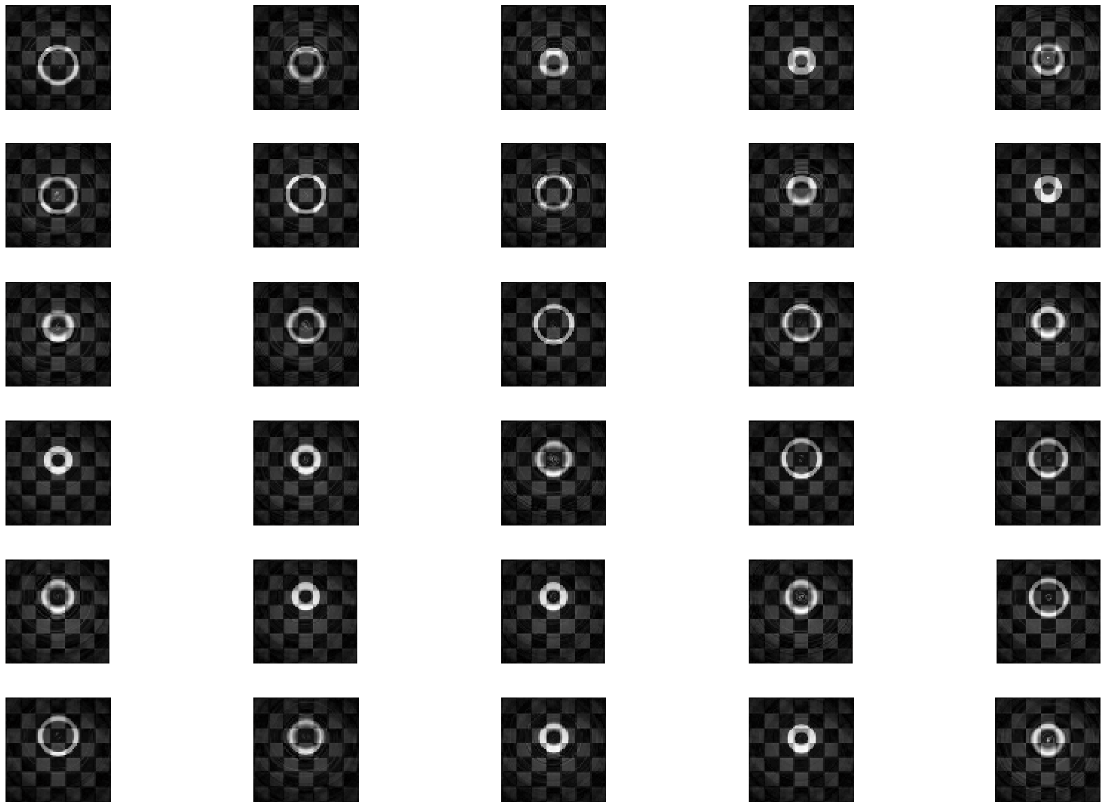


Figure 12. Original image sequence with static background

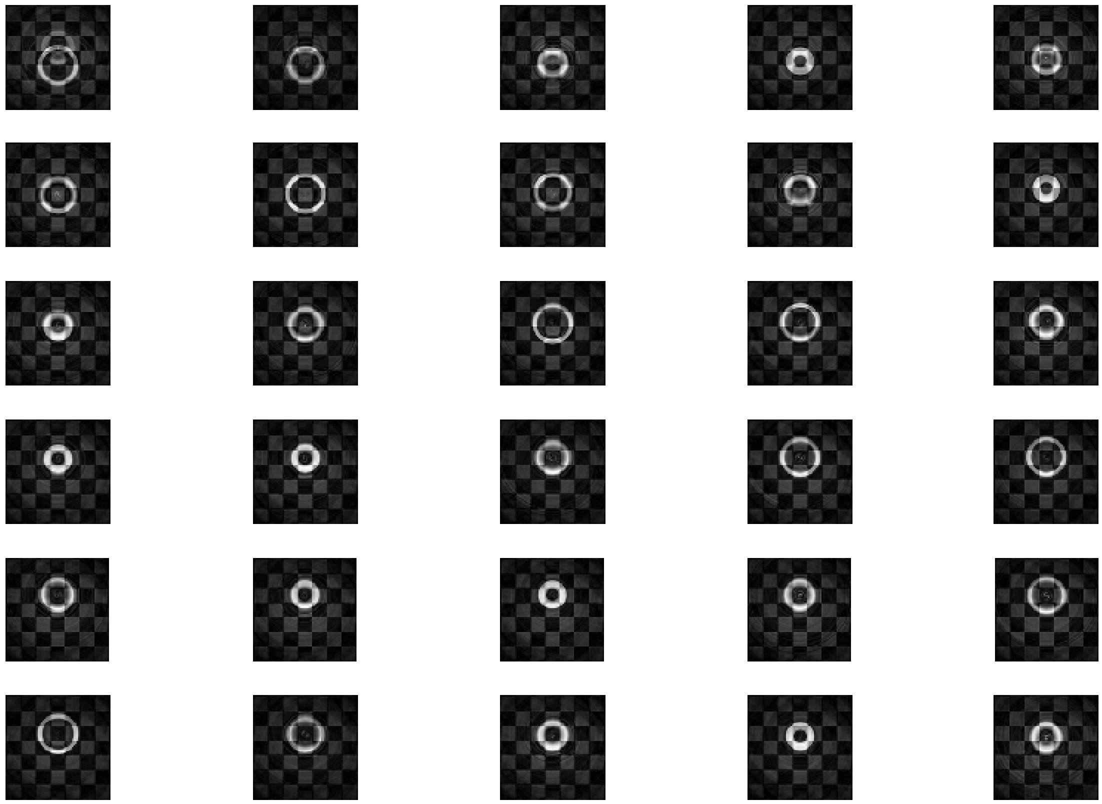
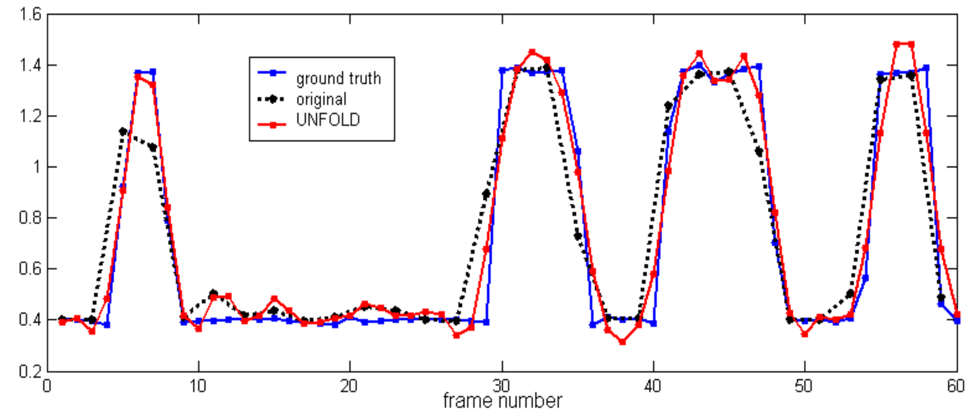


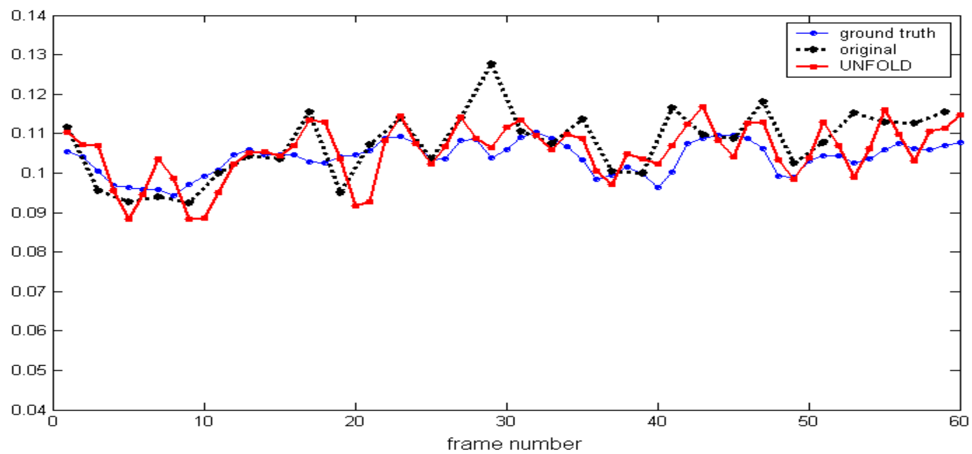
Figure 13. Image sequence obtained from UNFOLD

As the validation for the effectiveness of UNFOLD, I computed the temporal variation of the average value of small image block with the size of 3 x 3. Fig. 14 shows the temporal variation of the block at (60, 60) where the ring signal is dominant. I can see that the result from UNFOLD keeps track with the ground truth quite well. One side effect observed is the overshoot / undershoot at the abrupt change of signal which is caused by using rectangular window for filtering. But it's not significantly visible compared to the signal level and could be lessened by smoother window.

Another temporal variation is computed from the image block at different position (110, 120) which doesn't experience ring signal. Therefore the signal level is very low and the difference value between ground truth and UNFOLD is satisfactorily small.



(a) block at (60, 60)



(b) block at (110, 120)

Figure 14. Temporal variation of image blocks at different locations

Conclusion

From the analytic formulation and numerical simulation it can be rigorously proven that the underlying principle of UNFOLD is still valid in concentric ring trajectories. That is, when the even ordered rings and odd ordered rings are sampled alternatively, the phase of PSF's sidelobe will be inverted alternatively. I found that this phenomenon can carry over also to spiral trajectories from the simulation study of its PSF.

The dynamic phantom studies show that the image frames from UNFOLD are free from aliasing as desired, and their qualities are comparable to those of images reconstructed in the conventional way. Therefore using UNFOLD in spiral imaging, twofold increase in temporal resolution can be achieved compared to the conventional spiral imaging.

References

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