In this report, inversion recovery (IR) balanced SSFP with the $\alpha/2$ preparation pulse is simulated for the ideal case where the spin resonant frequency corresponds to the middle of the passband in the balanced SSFP steady-state signal profile. Simulation studies are presented to assess the effects of various choices of T1, T2, and flip angles on the transient signal responses of the IR balanced SSFP sequence. Moreover, after finding parameters that characterize the exponential behavior of IR transient signal curves, T1 and T2 values are analytically computed. Simulation results show that the computed T1 and T2 values are comparable to the true T1 and T2 values.
1 Introduction

Inversion recovery (IR) sequence is useful to produce images with high T1 contrast and null out a certain T1 species. Taken a step further, it is also effective to measure T1 values. Clinically, measuring T1 values is essential for better tissue characterization and quantification. In a simple 2DFT IR, a $180^\circ$ pulse is applied and after a specified time $T_I$, $90^\circ$ excitation pulse is turned on to acquire readout signals. Then, $M_z$, i.e., longitudinal magnetization, recovery time is taken until transverse magnetization components are fully decayed prior to the next inversion pulse. Therefore, typically repetition time (TR) is greater than a range of T2 values so that a large amount of scan time is required to obtain an entire image containing T1 values. To improve speed, the rapid imaging techniques with a shortened TR such as IR snapshot fast low-angle shot (FLASH) and IR steady-state free precession (SSFP) have been studied for T1 measurement, and IR balanced SSFP imaging was found to be more effective for T1 measurement than IR snapshot FLASH [1].

For the spin resonant frequencies whose steady-state signals are in the middle of the passband, it is well known that IR balanced SSFP with the $\alpha/2$ preparation method presented by Deimling and Heid [2] gives transient signal curves similar to longitudinal relaxation curves. Moreover, the transient signal curves depend on parameters T1, T2, and flip angles, so for a given flip angle the characteristics of the transient responses depend on both T1 and T2 values. Hence, the goal is to measure T1 and T2 values given the signal values of the IR transient responses and the imaging parameters such as flip angle, TR, etc. This report is organized as follows. First, the IR balanced SSFP sequence is described in section 2. In section 3, there is a brief review of the method presented
by Schmitt et al. [3], where T1 and T2 values are analytically found after obtaining parameters that fit the exponential functions characterizing IR transient signal curve. Finally, in sections 4 and 5, simulation results and discussion are provided.

2 Inversion Recovery Balanced SSFP

IR balanced SSFP sequence is illustrated in Fig. 1. First, the $180^\circ$ pulse is applied to invert the initial magnetization vector oriented to -z direction. After an elapse of a time TI, the $-(\alpha/2)_y$ pulse, whereby a spin is tipped by a flip angle of $-\alpha/2$ about y-axis, is applied followed by an elapse of a time TR/2, and then a train of $\alpha_x$ pulses are applied with a period of TR. In this scenario, the spin is assumed to undergo a phase accrual of $\pi$ radian over one TR in order to make the transient response of this IR sequence show a smooth curve like IR relaxation curves. Also, it is noted that the IR sequence described above performs in the same manner as the sequence where $\alpha/2_x$ preparation pulse is applied followed by a train of alternate polarity of $\alpha_x$ pulses with no phase accrual of spin. It is known that this $\alpha/2$ preparation method is simple in implementation and performs well in transient response for a “small” range of frequencies around the middle of the passband in balanced SSFP steady-state signal profile. As denoted by ACQ in Fig. 1, each k-space readout data is acquired in the middle between the excitation pulses. Since TR is very short, typically a few milliseconds, hundreds of k-space readout data can be acquired even with a single inversion pulse. Therefore, a rapid measurement of T1 values is possible with the IR balanced SSFP.
3 T1 and T2 Measurement

This section briefly reviews the method of T1 and T2 measurement presented by Schmitt et al. [3]. For the spin resonant frequencies whose steady-state signals are in the middle of the passband, the transient responses of the IR balanced SSFP with the $\alpha/2$ preparation can be described as the following exponential function:

$$s(t) = s_{st}(1 - INV \cdot \exp(-t/T_1^*)),$$

where the steady state signal $s_{st}$ is measured from the IR curve, and $INV$ is determined from the equation

$$INV = 1 + \frac{s_0}{s_{st}},$$
where $s_0 = M_0 \sin \frac{\alpha}{2}$. Finally, $T_1^*$ is obtained as a minimizer of the following least squares function $\Phi$:

$$\Phi(T_1^*) \triangleq \sum_{i=1}^{N} \left\{ s_{st}(1 - INV \cdot \exp(-t_i/T_1^*)) - s(t_i) \right\}^2,$$

where $N$ is a total number of the measured signal values and $s(t_i)$ is the signal value measured at time $t_i$. Since $\Phi$ is convex [4], it is straightforward to find $T_1^*$.

Once $T_1^*$ is found, it follows that $T_1$ and $T_2$ are easily computed by using:

$$T_1 = T_1^* \left[ \cos^2 \frac{\alpha}{2} + (A \cdot INV + B) \sin^2 \frac{\alpha}{2} \right]$$

$$T_2 = T_1^* \left[ \sin^2 \frac{\alpha}{2} + (A \cdot INV + B)^{-1} \cos^2 \frac{\alpha}{2} \right]$$

with

$$A = 2(1 - \cos \alpha)^{-1} \cos \frac{\alpha}{2}$$

$$B = (1 + 2 \cos \frac{\alpha}{2} + \cos \alpha)(\cos \alpha - 1)^{-1}.$$  

4 Simulation Results

Simulations were performed to assess the effects of $T_1$, $T_2$, and flip angles on the IR balanced SSFP transient responses. Moreover, $T_1$ and $T_2$ values were computed using the method presented in section 3 and then compared with true $T_1$ and $T_2$ values. The dynamics of spin magnetization vectors characterized by precession, relaxation, and excitation were implemented in a matrix form based on the Bloch equation. The signal values, which represent the magnitude of transverse magnetization vectors, were obtained in the middle between excitation pulses. Thus, the IR transient
Figure 2: IR balanced SSFP transient response curves: (a) Signals are absolute-valued, and (b) signals that precede a minimum signal value are inverted.

Signal curves would be of the form as shown in Fig. 2 (a). Since it is more natural to view the transient responses as increasing IR exponential functions, the signals that precede a minimum signal value were inverted as shown in Fig. 2 (b). Clearly, the IR balanced SSFP sequence was implemented in the same manner as illustrated in Fig. 1, where TI and TR values were chosen to be 10 ms and 5 ms, respectively. The reference true T1 and T2 values were provided under the assumption of the static $B_0$ field at 1.5 T.
4.1 Effect of Flip Angles on Signals

As can be seen from Fig. 3, for each different tissue, the IR transient responses were plotted separately for the flip angles of 10°, 30°, 50°, 70°, and 90°. It is observed that relatively high intensities of steady-state signals were attained for the flip angles of 30° and 50°. Unless otherwise noted, the flip angle of 50° would be used for the IR balanced SSFP sequence in what follows.

4.2 Effect of Flip Angles on IR Longitudinal Relaxation Curves

IR longitudinal relaxation curves were obtained by plotting longitudinal magnetization components with increasing time for the flip angles of 10°, 30°, 50°, 70°, and 90° when T1= 790 ms and T2= 92 ms (white matter) as shown in Fig. 4. As the flip angles increase, the longitudinal relaxation curves reach their steady-states faster, and the longitudinal values in the steady-states get lower. Meanwhile, it is seen that unlike all the IR balanced SSFP relaxation curves, the reference IR longitudinal relaxation curve has a full range of longitudinal recovery, which is -1 to 1.

4.3 Effect of T1 and T2 Values

First, we changed T1 values with a T2 value fixed, as shown in Fig. 5 (a). The smaller are T1 values, the steady-state signal levels are higher and the signals reach their steady states faster. On the other hand, we changed T2 values with a T1 value fixed. As can be seen from Fig. 5 (b), the steady-state signal levels are higher for a larger T2.
Figure 3: Transient responses of IR balanced SSFP with $\alpha/2$ preparation pulse for various flip angles. Illustrations are given for four different tissues, i.e., gray matter, CSF, liver, and fat.
Figure 4: IR longitudinal relaxation curves for flip angles of $10^\circ$, $30^\circ$, $50^\circ$, $70^\circ$, and $90^\circ$ when $T_1 = 790$ ms and $T_2 = 92$ ms (white matter).
Figure 5: IR balanced SSFP transient responses when T1 varies with T2 fixed and T2 varies with T1 fixed.
4.4 Effect of Changing T1 and T2 with Fixed T1/T2

It is well known that the steady-state signal is roughly proportional to T2/T1. In this simulation, for T1/T2 fixed to 3, T1 and T2 values were changed accordingly. As is seen from Fig. 6, all the three curves converge to the same steady-state signal value, but the time to reach the steady state increases with increasing T1 values.
4.5 T1 and T2 Measurement

Provided that the signal data were obtained from the IR transient responses, T1 and T2 values were computed based on the signal data and then compared with true T1 and T2 values. The details described herein were attempted to be similar to the experimental method by Schmitt et al. [3]. The steps are described as follows:

**Step 1:** Average the signal values from the first 21 consecutive ACQs, i.e., 21 phase encodes in 2DFT, comprising one image segment. Let this average be denoted by $M_1$. Likewise, $M_2$ denotes the average of the signal values from the next 21 ACQs, and so on. This averaging is reasonable since one image segment is obtained from 21 consecutive ACQs.

**Step 2:** Repeat **Step 1** for the next 37 image segments to obtain $M_2, M_3, \ldots, M_{38}$.

**Step 3:** Compute $INV$ using (2), where $M_{38}$ is chosen to be $s_{st}$ since it is closest to the steady-state signal value.

**Step 4:** Find $T_1^*$ that minimizes the convex function $\Phi$ in (3). Here, the golden search [5], which is effective for minimizing a convex function of one-dimensional variable, was used to find $T_1^*$ within 1 ms accuracy with 20 iterations.

**Step 5:** Compute T1 and T2 values using (4) and (5), respectively.

**Step 6:** Compare T1 and T2 values obtained from **Step 5** with true T1 and T2 values.
Figure 7: Illustration of the averaged signal values \( \{M_i\} \) (denoted by □) and the corresponding IR exponential curves fitted with the \( \{M_i\} \).

Fig. 7 illustrates the averaged signal values \( \{M_i\} \) obtained using **Step 1** and **Step 2** and the corresponding IR exponential curve whose parameters are fitted with the \( \{M_i\} \) using **Step 3** and **Step 4**, for the case when \( T_1 = 260 \) ms and \( T_2 = 85 \) ms. It is observed that the IR exponential function appears appropriately fit with the averaged signal values \( \{M_i\} \).

Moreover, as shown in Table 1, the \( T_1 \) and \( T_2 \) values computed using the steps described above are compared with the reference true \( T_1 \) and \( T_2 \) values, for the various tissues, i.e., gray matter,
Table 1: Comparison of measured T1 and T2 values with true T1 and T2 values.

<table>
<thead>
<tr>
<th>Tissue</th>
<th>T1</th>
<th>T2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>True(ms)</td>
<td>Measured(ms)</td>
</tr>
<tr>
<td>Gray matter</td>
<td>920</td>
<td>933.54</td>
</tr>
<tr>
<td>White matter</td>
<td>790</td>
<td>802.37</td>
</tr>
<tr>
<td>CSF</td>
<td>2000</td>
<td>2028.4</td>
</tr>
<tr>
<td>Liver</td>
<td>490</td>
<td>511.92</td>
</tr>
<tr>
<td>Fat</td>
<td>260</td>
<td>264.90</td>
</tr>
</tbody>
</table>

white matter, CSF, liver, and fat. For all the tissue types considered, the T1 and T2 values are both in agreement with the true values within percentage error of 5%.

5 Discussion

In all the simulation results considered, it was assumed that the spin frequencies correspond to the middle of the passband in the balanced SSFP steady-state signal profile. In reality, the spins would have a wide range of phase accrual over one TR due to off-resonance effects such as field inhomogeneities, magnetic susceptibilities, chemical shifts, flow effects, etc. Hence, IR transient response curves are expected to be different for spins that have different phase accruals over one TR. Subsequently, it would lead to poorer measurement of T1 and T2 values if one would consistently
use the method of measuring T1 and T2 values presented here. Therefore, it would be necessary to develop a method that is robust in measuring T1 and T2 values even for a wide range of phase accruals due to off-resonance effects.

Moreover, with the $\alpha/2$ preparation pulse, the IR transient responses manifested “pure” exponential curves as shown in the simulation results. However, it is well known that balanced SSFP has oscillatory transient behaviors due to the off-resonance effects before the signals reach their steady-states. In the simulation studies, ideally the “pure” IR exponential curves were obtained, so that the signal values at the early stage of IR curves were used to measure T1 and T2 values. However, these signal values would not be reliable due to the oscillatory transient responses in reality, so they might be discarded for the better measurement of T1 and T2 values. In other words, this loss of reliable data would lead to a poor performance of measuring T1 and T2 values. Consequently, it would be important to develop a IR balanced SSFP that reduces the oscillatory transient behavior for a wide range of off-resonance frequency band.

References


