Reconstruction from Arbitrary and Partially Sampled k-space Trajectories

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Introduction

Reconstruction of MRI images from k-space sampling along arbitrary trajectories and partial sampling of k-space have number of advantages in fast imaging. Non - Cartessian sampling trajectories such as spiral, PR, Lissajous, rossete, stochastic etc. have different advantages in different applications. For example, it has been observed that the motion artifact is less severe in spiral and PR trajectories. These trajectories don't show a systematic shifting or blurring off-resonance artifacts like 2DFT but rather a random distribution of off-resonance signal over the whole off-resonance image. Spiral MRI has grown popular in number of clinical applications including abdominal imaging, lung imaging, functional MRI, angiography and others.

Gridding is a general method for fourier inversion of the irregularly sampled data. In this report I study a fast iterative algorithm for reconstruction of data from a non-uniform sample trajectory.

In addition to implementing [1,2,3], I investigated the following:
An important technique of reduction of scan-time is partial sampling of the k-space. In this report I propose to use the symmetry property of the spiral trajectory and illustrate that the spiral k-space can be under sampled by a factor of 2 and rest of the data for alternate spirals can synthesized using the symmetry properties of the fourier transform.

Iterative reconstruction from from Arbitrary Samples[2]

The relation between MR signal and the image can be modeled as

$$I(\vec{r}) = \int_{FOV} s(\vec{k}) \exp(2i\pi \vec{k}\vec{r}) d\vec{k}$$

where s(k) denote the k space data and I denote the image. After discretization, it can be written as

$$I(\vec{k}) \approx \sum_n s(\vec{k}_n) \exp(2i\pi \vec{k}_n \vec{r}_n) \Delta\vec{k}_n$$

An important step in gridding and DrFT algorithms is the calculation of the sampling density $\Delta\vec{k}_n$. These weighting coefficents determine how well the integral above is computed. The sampling density can be computed by using voronoi diagram or Pipe and Menon methods. However these give only approximattions and are source of errors in the computation of above integrals.

In this project I investigate an alternative approach which is based on the inverse model[2].

The fourier inversion theorem gives

$$s(\vec{k}) = \int_{FOV} I(\vec{r}) \exp(-2i\pi\vec{k}\vec{r}) \, d\vec{r}$$

After discretization of this equation, we get

$$s(\vec{k}) \approx \Delta a \sum_n I(\vec{r}_n) \exp(-2i\pi\vec{k}_n\vec{r}_n)$$

The key point here is that as opposed to DrFT, this discretization is in image domain and hence Δa is constant. Therefore no density compensation is required in the iterative approach. We represent above equation in matrix vector form as

$$s = H i$$

Where H is called system matrix. Solution of this system of equations is usually ill posed due to extreme sensitivity to high frequency perturbations and therefore discretization of this integral gives rise to ill conditioned system matrix. However, behavior and the solution of such inverse problems is very well studied topic and many iterative algorithms exists which are robust and fast. Here I use conjugate gradient method for solving the system of equations. In connection to discrete ill-posed problems, the CG iterations are stopped long before the final convergence. The main reason for this is the intrinsic regularizing effect of the CGLS iterations applied on the normal equations. The method produces iteration vectors in which the spectral components associated with the large eigenvalues of the system matrix converge faster than the remaining components.

I applied this algorithm to a 64x64 image. I also implemented Direct Fourier transform algorithm [1] to compare it with this algorithm. I tested these algorithms for PR, uniformly sampled spiral and Lissajous trajectories. I found that the iterative algorithm requires a large amount of memory, but it is more robust to noise because of the regularization effect described above. This can be less significant since noise levels in MR data are usually small. There is another advantage of the iterative algorithm which is that it does not require density compensation as opposed to the DrFT algorithm and therefore is more robust to these errors. Both these algorithms show small runtime of ~.68 sec and ~.79 sec respectively (not including the one time computations of system matrices).
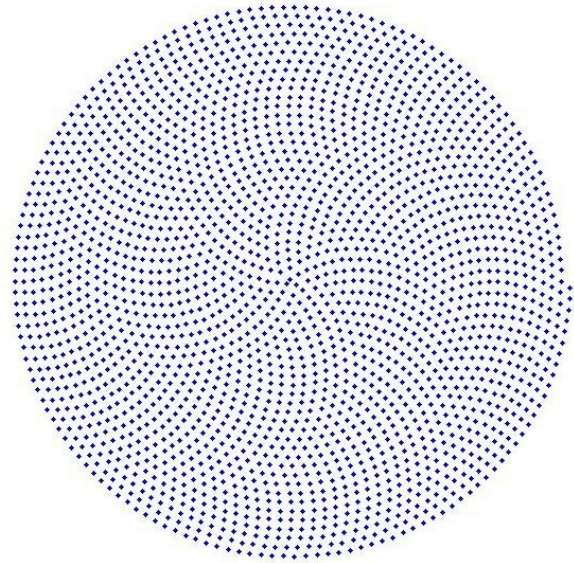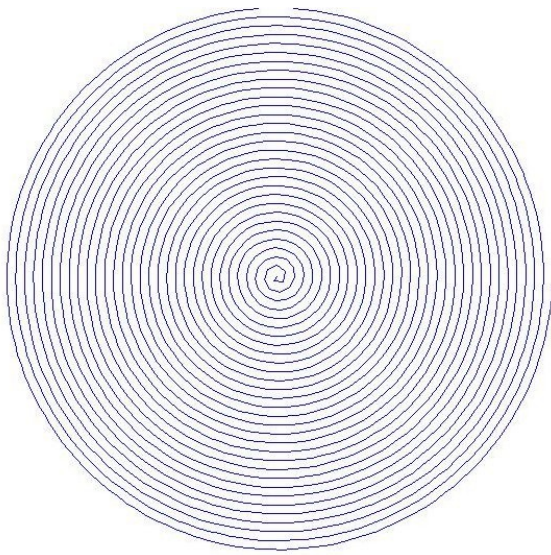
Figure 1: Uniformly sampled spiral. Sample points lie at equal distances according to arc-length. It can be seen that sampling density is approximately same at low as well as high frequencies.
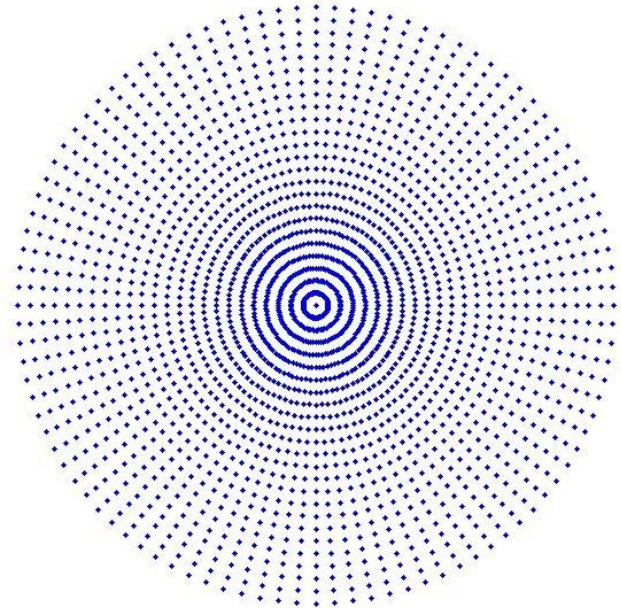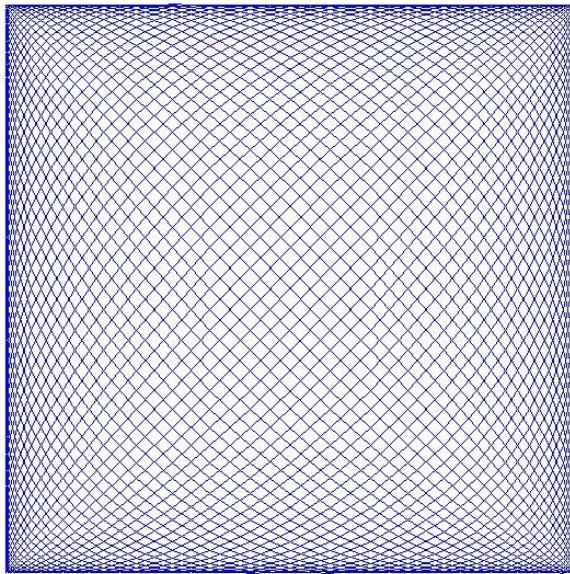
Figure 2: Lissajous and PR trajectories.
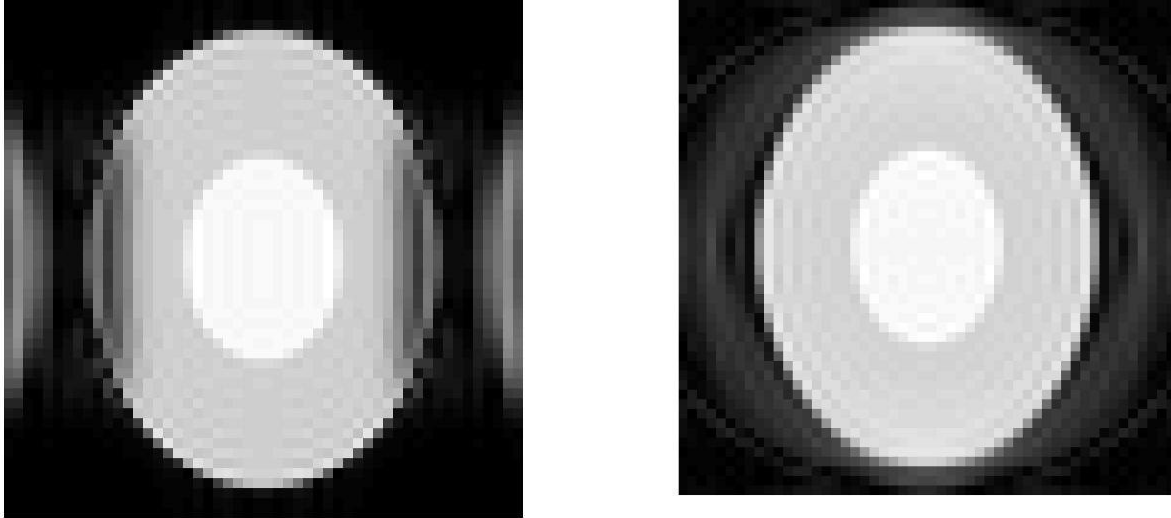
**Figure 3:** Reconstruction with Lissajous and PR trajectories.

Lissajous trajectory required a lot more samples than the spiral trajectory. I found spiral trajectory most efficient in terms of number of samples required and hence the scan time and memory requirement for the system matrix. Hence, I used variable density and uniform spirals for rest of the project even though the techniques demonstrated here can be used for arbitrary trajectories.

### Using Segmentation along with iterative reconstruction

We combine the iterative reconstruction with a segmentation approach similar to the one presented in [3] which uses the fact that in many medical images, there are black areas surrounding the object. We can take advantage of this fact to reduce the support (pixels to be reconstructed). This reduces size and improves condition number of the system matrix. We use the fact that in spiral/PR/Lissajous trajectories, aliasing images are scattered around the original object and hence it is possible to detect the support of the object by detecting long edges. We do this in following steps:

1. Reconstruct the image iteratively by using the method presented above
2. Apply sobel edge detector on the image to find the central main object
3. apply morphological operations of dilate and flood fill to fill up the holes to create an object mask
4. Delete columns in the system matrix corresponding the zero pixels in the mask.
5. Reconstruct the image again only for the mask pixels, reorder to put them in appropriate locations.

I present the results first for a simple phantom and then real data obtained from class website.
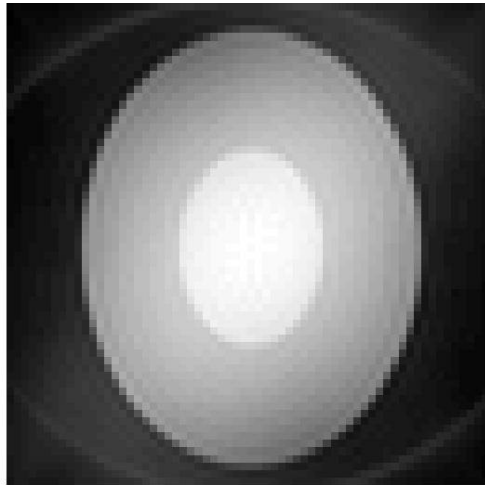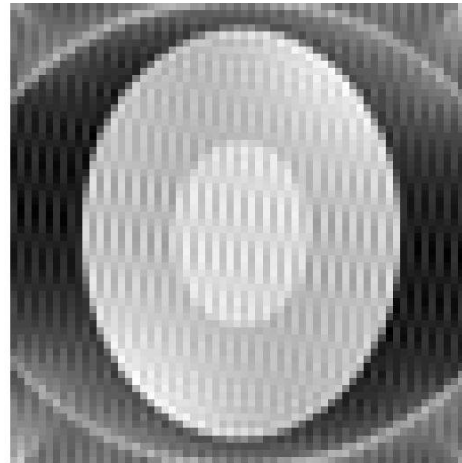
<div align="center">Gridding   DrFT</div>

**Figure 4:** Reconstruction by gridding and DrFT.

Sample density is estimated by voronoi diagrams. In order to avoid low frequency artifact, the low frequencies should be densely sampled. In case of uniform spial, this is not the case resulting in inaccuracies in density estimates of low frequencies. The low frequency artifacts are visible in both the cases. For DrFT, density estimates of high frequencies cause high frequency artifacts as well. Since iterative method does not require density estimation, these problems can be avoided. Also note that aliasing can be seen in both the cases. However it is much more conspicuous in DrFT case.
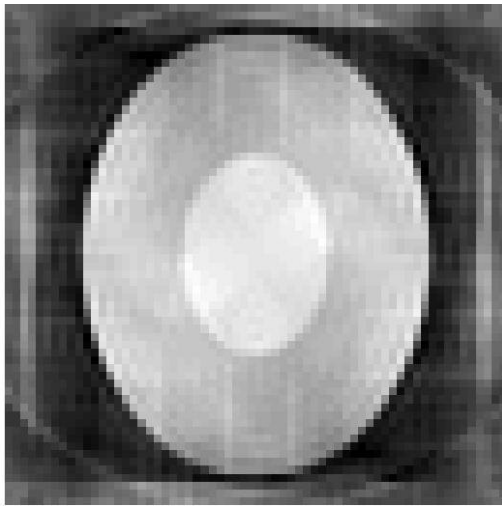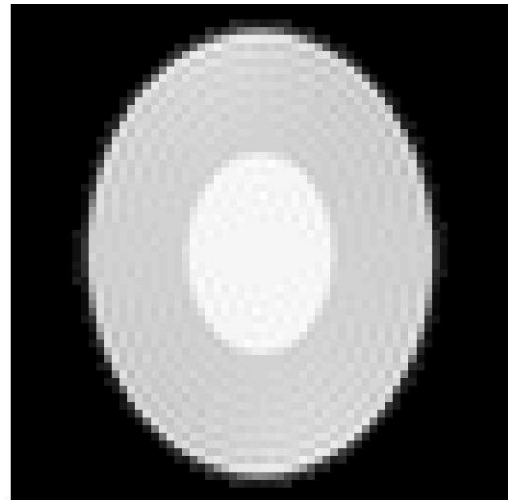
(a)
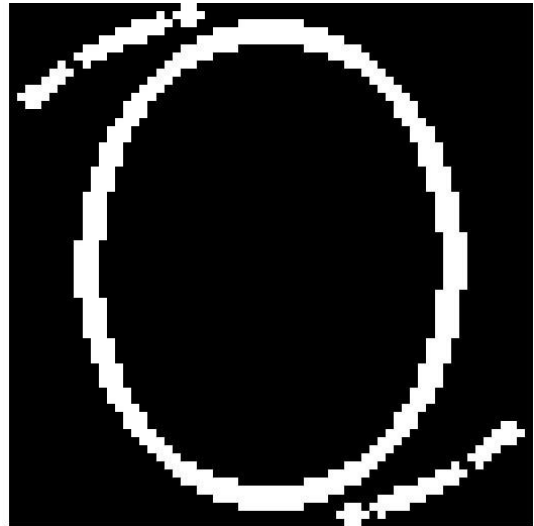
(b)
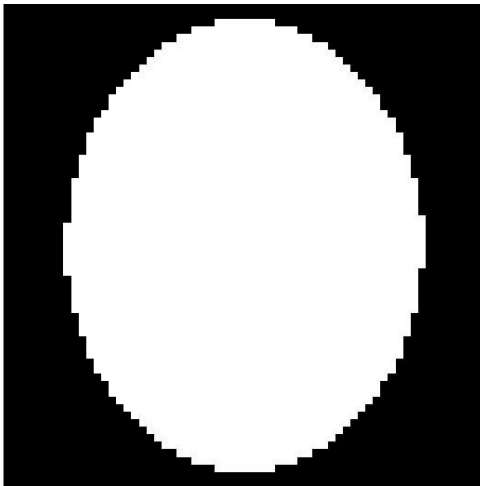
(c)

(d)

Figure 5: Reconstruction by iterative technique.
(a)We stop the iterations after only 2 steps obtaining very rough estimate of the support of the object. (b)Then we apply sobel edge detector to extract the major edges. (c) Then we extract the mask using morphological operations of erode, dilate and fill. This corresponds to support of the object. Then we delete the columns in system matrix H corresponding to black pixels and reconstruct only the pixel which lie inside the mask. This improves the condition number of matrix resulting in faster convergence in 5 iterations and much better reconstruction. (d) Using this technique we could reconstruct the image with a sparsely sampled trajectory (32 turn spiral, 64x64 image) and also this technique avoided aliasing. Also no density estimation was required avoiding aliasing and artifacts seen in previous figure.

In second set of experiments, I used data from the class website for variable density spiral. The data given is for 128x128 image. But due to memory requirement for the system matrix I could reconstruct only 64x64 images. To create data for 64x64 image, I first multiplied k-space data by 2 and then took central half of the k-space.
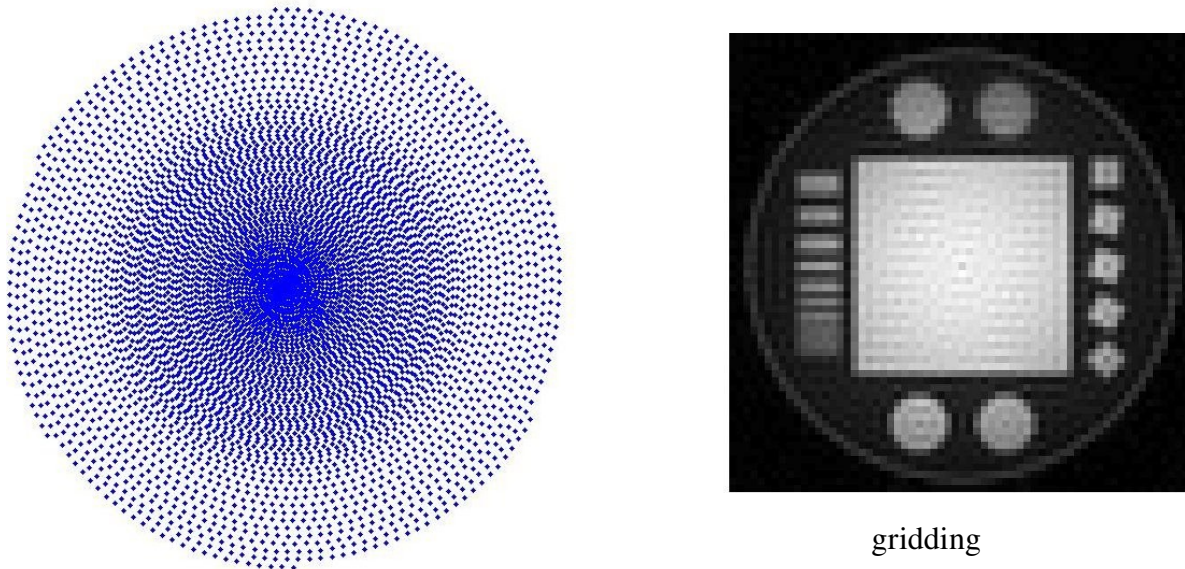
gridding

Figure 6: Samples from non-uniform spiral trajectory. Reconstruction by gridding. We can see high frequency artifact. This is due to the fact that high frequencies are sparsely sampled.
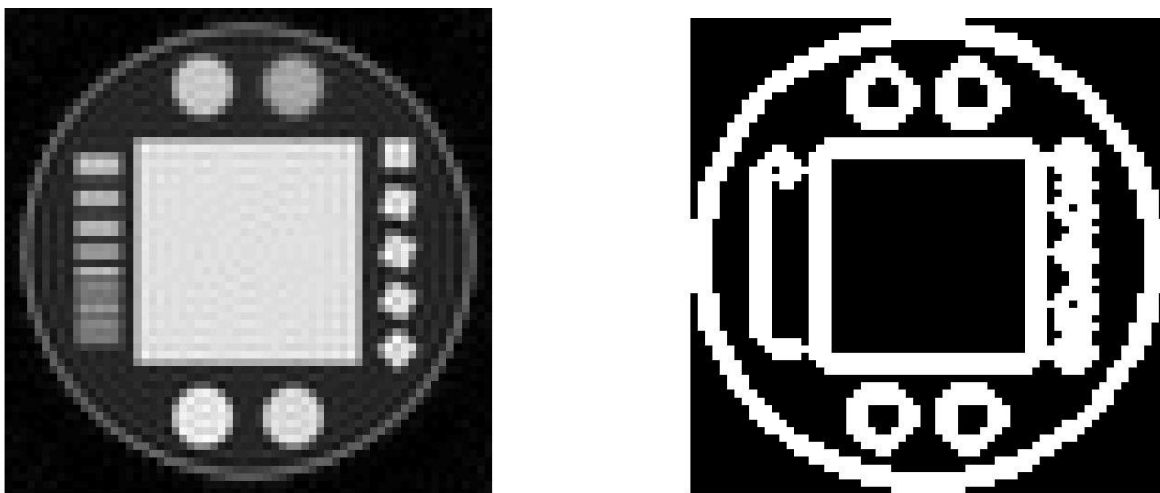
Figure 7: Reconstruction after 2 iterations of the iterative algorithm. The second figure shows edge map extracted by sobel edge detector.

Figure 8: Morphological operations of erode, dilate and fill are applied to get the object mask representing the object support. After deleting the columns corresponding to blank pixels from the system matrix H, the image is reconstructed again iteratively for 5 iterations. We can see that the high frequency artifact is less severe (central square). Also the two disks on the top are better reconstructed resulting in better reconstruction compared to gridding.

Partial k-space Acquisition

In this section I use idea similar to the one discussed in class for PR trajectory. Here I apply that to spiral trajectory. In theory, most MRI images depict the spin densities as a function of position and hence should be real valued. Although this is not strictly true in practice, the phase of the spin densities varies relatively slowly and hence can be recovered from a low resolution image. This allows us to exploit the symmetry properties of the fourier transform. We know that for real valued signals, their fourier transforms have conjugate symmetry,

$$\vec{d}(k_x, k_y) = conj(\vec{d}(-k_x, -k_y))$$

Consider a real valued object. For spiral trajectories starting 180 degree apart, the k-space data along those trajectories must be identical. This is depicted in the following figure.
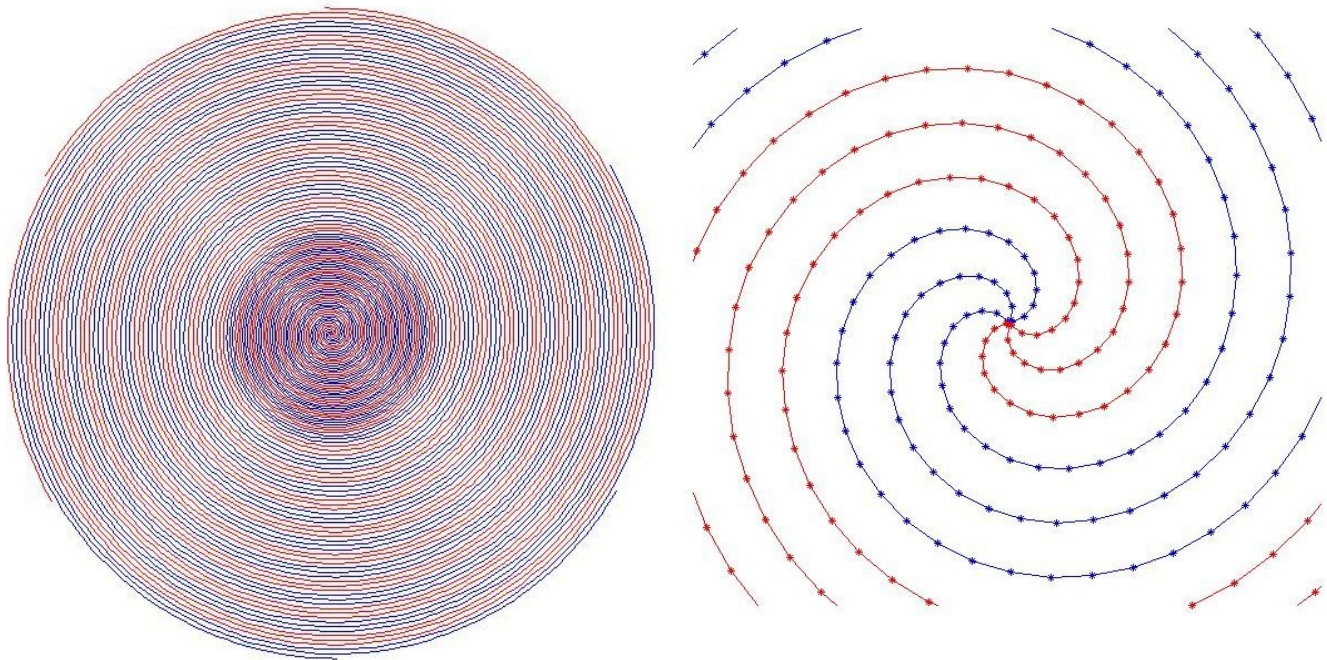
Figure 9: Symmetries of the spiral trajectories are shown above. If the object is real valued, then we only need to sample the blue spirals and data on red spirals can be synthesized by taking conjugate of data from diametrically opposite points on the blue spirals. The fact that the variable density spiral is densely sampled near the center allows us to reconstruct a low frequency phase map for the same FOV.
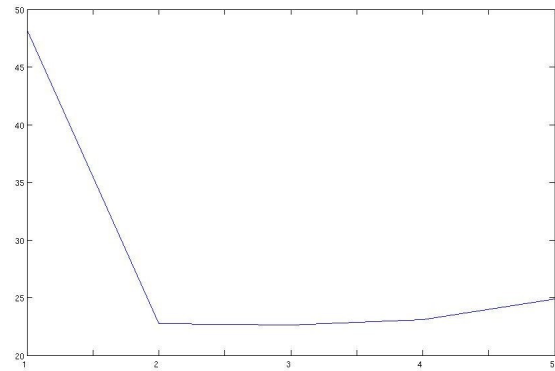
POCS algorithm

I used the following POCS algorithm for spiral trajectory.

1. Compute the system matrix H
2. Compute the synthetic k-space data on red spirals by taking conjugate of the given data on the corresponding blue spirals.
3. Iteratively reconstruct the image by solving the linear system of equations.
4. Iteratively reconstruct a low resolution image using central quarter of the spiral.
5. Estimate phase of the object from the low resolution image from step 4.
6. Create the new data d= H*(img*(exp(-i angle))) on the spiral trajectories.
7. Replace the data in blue spirals by the original data.
8. Goto step 3.

I observe that the algorithm converges in 4-5 steps and the phase estimate stabilizes to a small value.
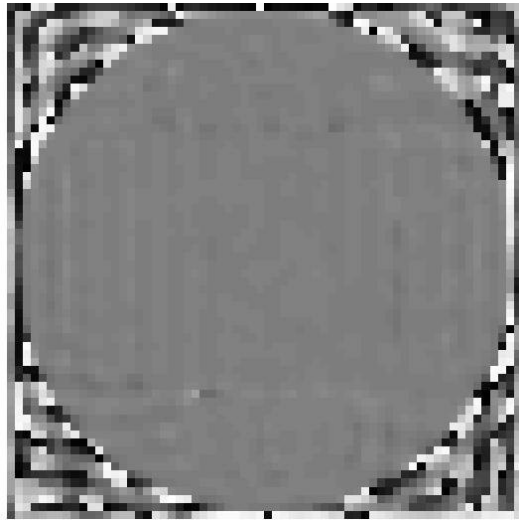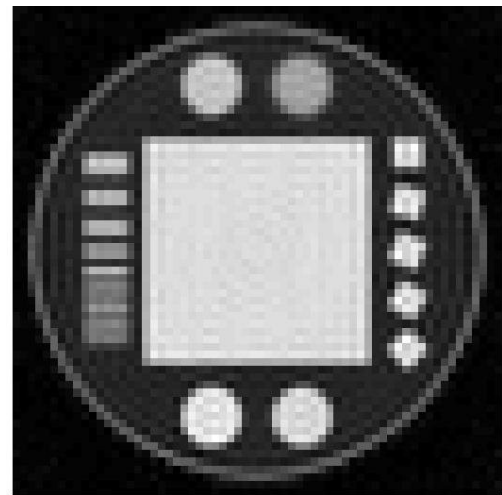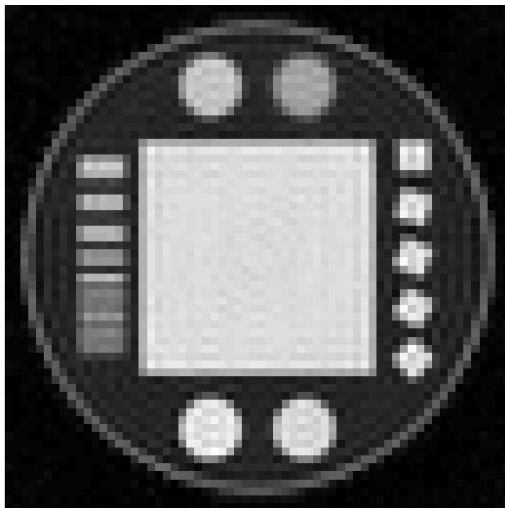
(a)

(b)

Figure 10:(a) Phase estimate of the image at the beginning. (b) Norm of the phase angle vs iteration number. Small rise at the end is mainly due to phase of zero.

Figure 11: Image reconstructed from partial k-space data and after using POC algorithm mentioned above. Although there is no noticeable change visible to the human eye since the phase was very small. The phase correction did take place as shown in graph in previous figure and algorithm converged.

To make this algorithm better, I used reconstruction using mask as explained in previous section,

instead of iterative reconstruction. ie. Step 3 in POCS algorithm above is replaced by

3a. Iteratively reconstruct the image.
3b. Detect the object support using sobel edge detector and morphological operations.
3c Delete the columns in system matrix corresponding to the zero pixels and reconstruct only the pixels lying inside the mask.

The exact same procedure can be applied to DrFT algorithm only difference is that we have to delete rows in the FT matrix and thus reconstructing only the pixels lying inside the mask. However this is equivalent to applying the mask on the reconstructed image directly hence it has no particular advantage.
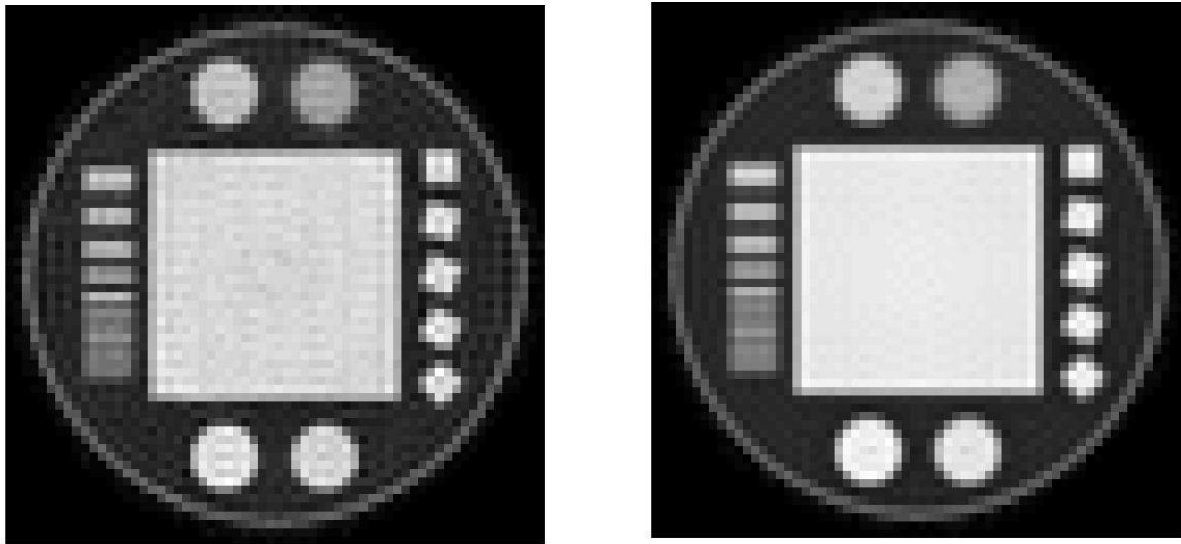
Figure 12: reconstruction by POCS algorithm for variable density spiral, with mask. The figure on the left shows DrFT reconstruction with mask applied. Figure on the right shows iterative reconstruction with mask applied. In both the cases, data from blue spirals was used to synthesize the data on red spirals. After that POCS algorithm was applied.

Conclusion

Reconstruction from arbitrary k-space trajectories such as spiral, PR are useful due to fast data acquisition. In this project, I implemented and investigated some techniques and their combinations for reconstruction from arbitrary trajectories.

I implemented iterative reconstruction which involves solving the inverse problem. While memory requirement for this algorithm is high, the reconstruction time is almost same as DrFT algorithm. Additionally by detecting the object support, we can reduce the size of the system matrix and improve its conditioning. I investigated these algorithms for uniform spiral, variable density spiral, Lissajous and PR trajectories. Among these spirals showed best quality reconstructions. I also implemented partial k-space reconstruction algorithm for variable density spiral trajectory similar to the idea for PR trajectory discussed in class. These methods can be used for acquisitions with very small scan time.

References

1. A. Maeda et al. *Reconstruction by Weighted Correlation for MRI with Time-Varying Gradients,* IEEE TMI 7(1) Mar 1988 pp 26-31.

2. B. Desplanques et al. *Iterative Reconstruction of Magnetic Resonance Images From Arbitrary Samples,* IEEE Trans. on Nuclear Science, 49(5) Oct 2002, pp 2268-2273.

3. H. Pirsiavash et. al. *An Iterative Approach for Reconstruction of Arbitrary Sparsely Sampled Magnetic Resonance Images,* IEEE Symposium on Computer-Based Medical Systems 2005.