

Velocity k-space analysis of Flow Effects in Echo-Planar, Spiral and Projection Reconstruction Imaging

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Introduction

Flow and motion in the object causes distortion in the MRI signal that is received. The effects of flow can be classified as (1) time of flight effects that are produced when there is motion in the object between excitation and readout and depend on the RF excitation and (2) phase effects that are caused by motion during readout and depend on the applied gradient waveforms. In this project, phase effects of flow are studied for different types of gradient waveforms, using a velocity k-space analysis [1]. The objective is to account for the difference in performance in flow imaging of different sampling trajectories, using the velocity k-space analysis. The following trajectories are considered:

- (1) Single-shot echo planar
- (2) Multi-shot spiral
- (3) 2D Projection Reconstruction (2DPR)

The effect of uniform and parabolic flow in a cylindrical vessel at different orientations is studied. A point spread function for flow is also obtained by imaging a small square object moving at uniform velocity.

Velocity k-space analysis:

The effect of flow during readout causes additional phase in the signal. The signal equation can be modified to incorporate this additional phase. Let $\mathbf{r}(t)$ be the position of the object at time t , $m(\mathbf{r}(t))$ be the object at that position, $\mathbf{G}(t)$ be the vector of gradient waveforms and $s(t)$ represent the signal obtained at t . The signal equation is then given by

$$s(t) = \int m(\mathbf{r}(t)) e^{-2\pi j \int_0^t \frac{\gamma}{2\pi} \mathbf{G}(\tau) \cdot \mathbf{r}(\tau) d\tau} d\mathbf{r}$$

For constant velocity flow, $\mathbf{r}(t)$ is given by

$$\mathbf{r}(t) = \mathbf{r}_0 + \mathbf{v}t$$

Then the signal equation incorporating the effect of flow is given by

$$s(t) = \int \int m(\mathbf{r}(t)) e^{-2\pi j \left(\int_0^t \frac{\gamma}{2\pi} \mathbf{G}(\tau) \cdot \mathbf{r}_0 d\tau + \int_0^t \frac{\gamma}{2\pi} \mathbf{G}(\tau) \cdot \mathbf{v} \tau d\tau \right)} d\mathbf{r} d\mathbf{v}$$

Now let

$$\mathbf{k}_r(t) = \int_0^t \frac{\gamma}{2\pi} \mathbf{G}(\tau) d\tau$$

$$\mathbf{k}_v(t) = \int_0^t \frac{\gamma}{2\pi} \mathbf{G}(\tau) \tau d\tau$$

Then, the signal equation can be expressed in terms of spatial and velocity k-space as

$$s(t) = \int_{\mathbf{r}} \int_{\mathbf{v}} m(\mathbf{r}(t)) e^{-2\pi j(\mathbf{k}_r(t) \cdot \mathbf{r}_0 d\tau + \mathbf{k}_v(t) \cdot \mathbf{v} d\tau)} d\mathbf{r} d\mathbf{v}$$

This can be viewed as a six dimensional Fourier transform.

$$s(t) = M(\mathbf{k}_r(t), \mathbf{k}_v(t))$$

The velocity k-space vector $\mathbf{k}_v(t)$ is a function of the spatial k-space variable $\mathbf{k}_r(t)$. While the former is the zeroth moment of the gradient waveform, the latter is the first moment. In 2 dimensions, let

$$\mathbf{k}_r(t) = \begin{bmatrix} k_x(t) & k_y(t) \end{bmatrix}$$

$$\mathbf{k}_v(t) = \begin{bmatrix} k_u(t) & k_v(t) \end{bmatrix}$$

then the data can be expressed as

$$\begin{aligned} s(t) &= M(k_x(t), k_y(t), k_u(t), k_v(t)) \\ &= M(k_x(t), k_y(t), k_u(k_x(t), k_y(t)), k_v(k_x(t), k_y(t))) \end{aligned}$$

The image can then be reconstructed from the data by taking an inverse 2DFT of the obtained data samples. In this project, 2D imaging is considered for all the analysis.

Object model

The k-space formulation of velocity directly leads to a model that incorporates motion in the object. The object can be represented in 4 dimensions, incorporating velocity in the x and y directions. For an object $m_s(x,y)$ moving at constant velocity with x and y components given by u_0 and v_0 , the object can be represented as

$$m(x,y,u,v) = m_s(x,y) \delta(u - u_0, v - v_0)$$

Parabolic flow in a cylindrical pipe can also be modeled with this approach by considering the projection onto the imaging plane (x-y plane in this case,) of a range of velocities. The velocities for parabolic flow are constant along concentric circles in the pipe and the maximum velocity occurs at the center. The parabolic velocity object model for a cylindrical vessel of radius R and maximum velocity u_m , with flow along the x (or u) axis is given by

$$m(x,y,u,v) = \delta(v) \frac{R^2}{u_m} \frac{1}{((1 - u/u_m)R^2 - y^2)^{1/2}},$$

$$\text{for } 0 < u < u_m, |y| < ((1 - u/u_m)R^2)^{1/2}$$

This is shown in Figure 1.

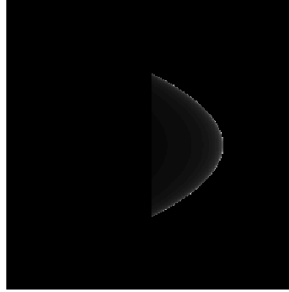


Figure 1: Parabolic flow along x axis.

k-space trajectories:

Phase effects depend on the gradient sequence that is used during readout. The trajectory traversed in spatial k-space has a corresponding trajectory in the velocity k-space, which determines the amount of distortion in the signal due to flow. The imaging sequences considered in this study of flow effects are echo planar imaging (EPI), multi-shot spiral imaging and 2D projection reconstruction (2DPR) [2].

1) Echo Planar Sequence: EPI is a fast imaging sequence in which multiple raster lines are acquired per excitation. After each readout along k_x , the trajectory blips upward in the k_y direction and covers the k-space. The gradient waveforms and corresponding k-space trajectory are shown in Figure 2 for a single shot EPI sequence with a readout of 39.5 ms and 16 phase encodes. The velocity k space values computed by finding the first moment of the gradient waveforms are shown in Figure 3, as a function of k_x and k_y .

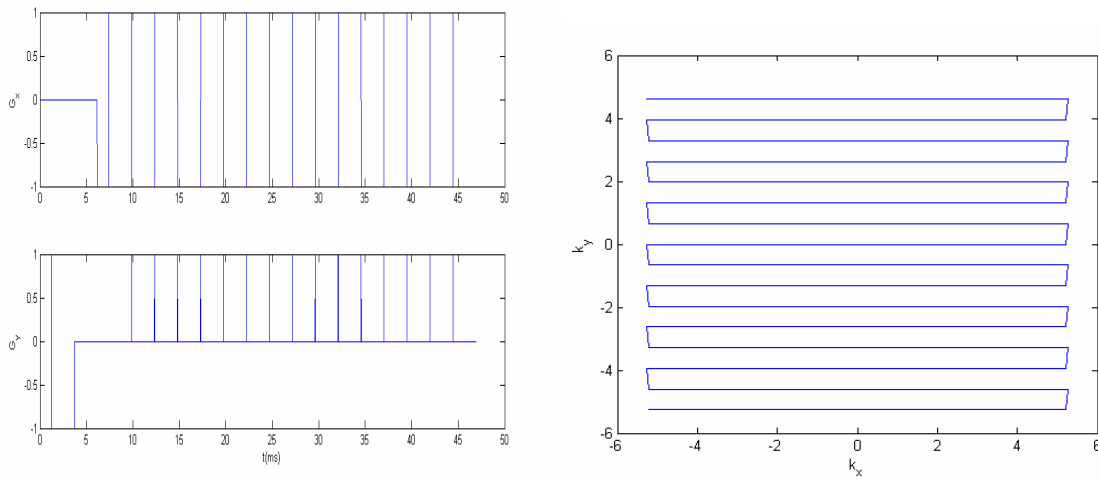


Figure 2: EPI imaging sequence and the corresponding k-space trajectory

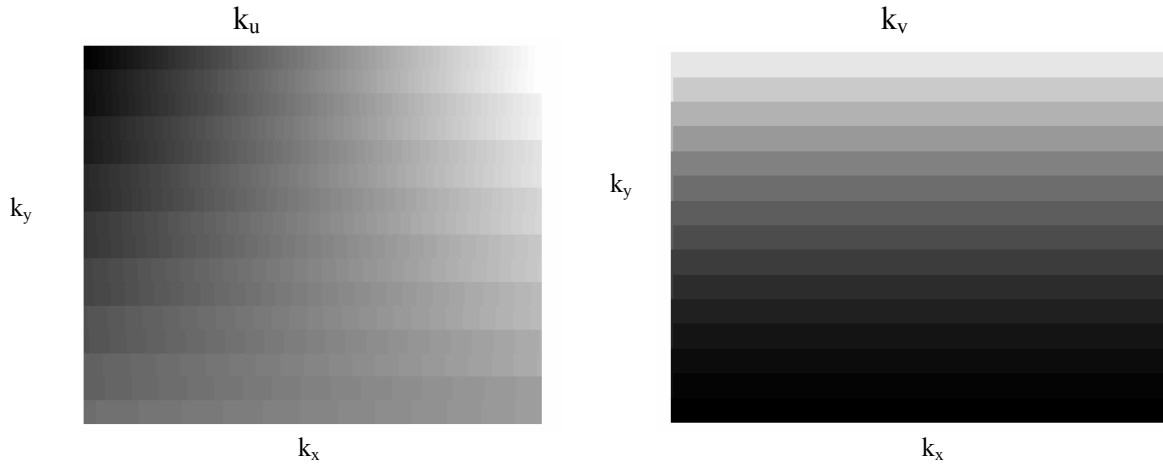


Figure 3: k_u and k_v as a function of spatial frequencies k_x and k_y for the EPI sequence.

As the readout gradient scans the spatial k-space at progressively later times, k_u increases, which can be seen in Figure 3 as lines of increasing intensity from left to right. The discontinuities in k_u along the k_y direction are due to the blip in the y direction, and can be made to smoothly vary by interleaving or by denser sampling in the y direction. Velocity k-space values for a single shot EPI sequence with 256 phase encodes and a total readout time of 39.5ms are shown in Figure 4. The resulting field of view (FOV) is 25.6 cm with a resolution of 1mm. The variation of k_u and k_v in the k_y direction is now smooth. The figure also shows the magnitude and phase of the vector $[k_u \ k_v]$, which do not change smoothly around the origin. The magnitude is least at the center, because of flow compensation in the gradient waveform and increases away from the origin.

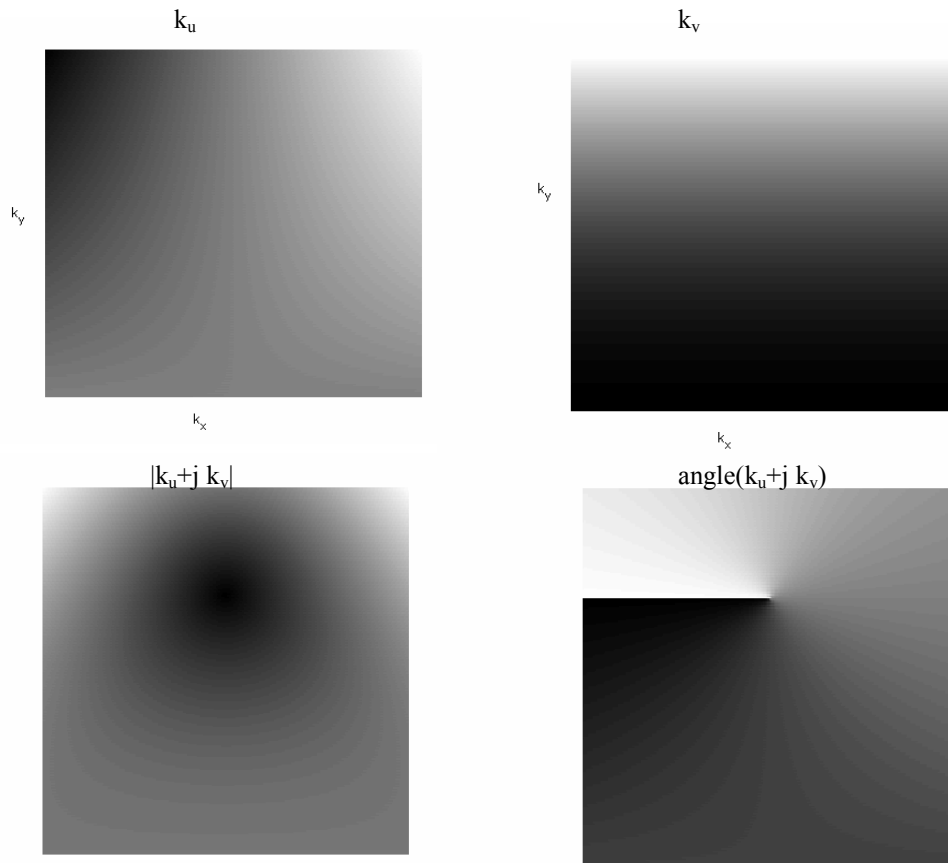


Figure 4: (Top row) : k_u and k_v as a function of spatial frequencies k_x and k_y for a 256 phase encode single shot EPI sequence. Bottom row: Magnitude and phase of the $[k_u, k_v]$ vector.

(2) Spiral Sequence:

In this sequence, the k -space data is sampled by starting at the origin and gradually spiraling outward with constant velocity. The gradients for one interleaf of a 16 interleave multi-shot spiral imaging sequence for a readout of 30.1ms are shown in Figure 5 along with the corresponding k -space trajectory. The gradients for the rest of the interleaves are obtained by rotating the given gradients by a small angle and repeating the process. The velocity k -space trajectory and the length and direction of the $[k_u, k_v]$ vector for this sequence are shown in Figure 6. The velocity k -space values are radially symmetric. The magnitude of $[k_u, k_v]$ vector is small around the origin and increases radially away from the origin. The direction of this vector varies from $-\pi$ to π , symmetrically around the origin.

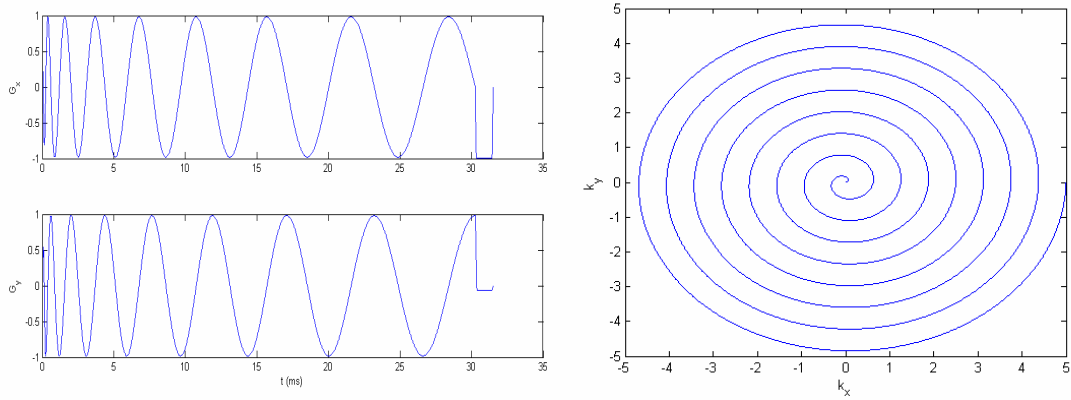


Figure 5: Spiral Sequence: Gradient waveforms and corresponding k-space trajectory.

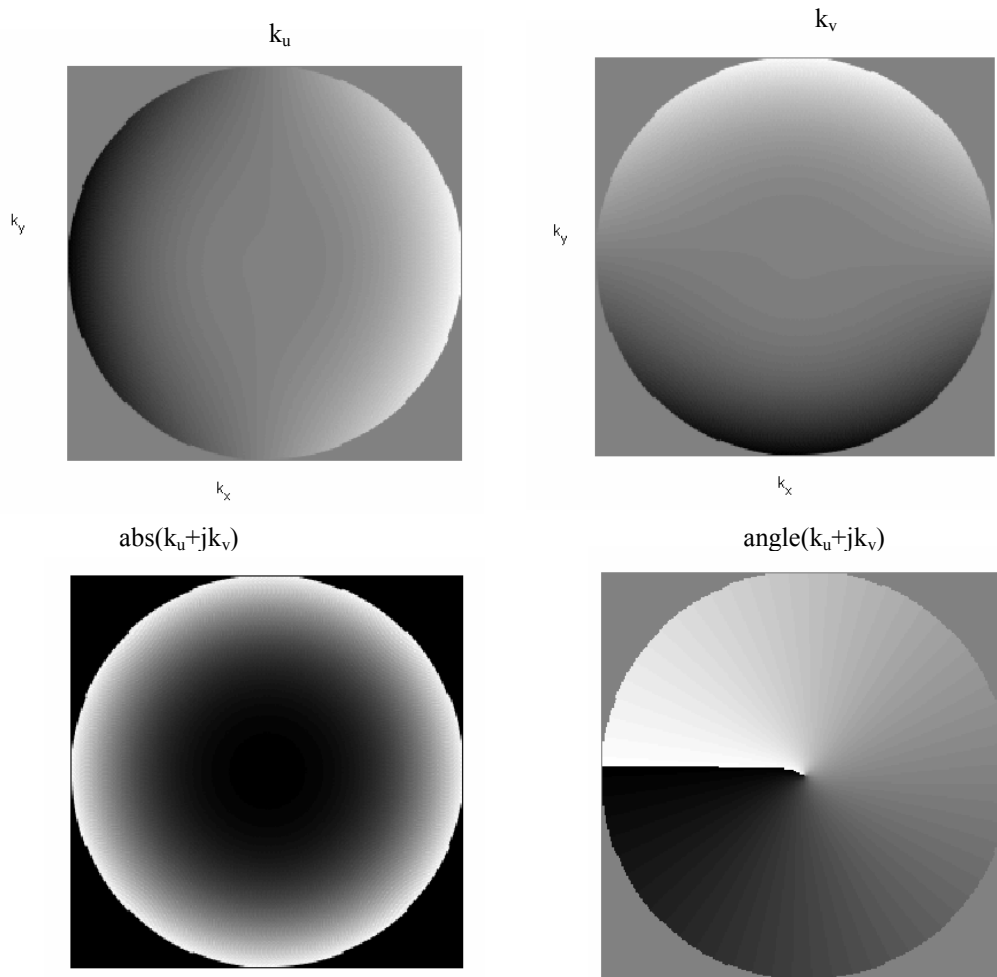


Figure 6: (Top row) k_u and k_v as a function of spatial frequencies k_x and k_y for a 16 interleave multi shot spiral sequence. Bottom row: Magnitude and phase of the $[k_u \ k_v]$ vector.

(3) 2D Projection Reconstruction Sequence: In this sequence, the trajectory starts at the origin and travels along a radial spoke at a constant angle. One spoke is traversed during

each readout cycle, hence sampling on a circular grid. The gradient waveforms for 10 radial spokes with a readout of 39.5 ms is shown in Figure 7a and the spatial k-space trajectory for 256 spokes is shown in Figure 7b. The velocity k-space trajectories, with the length and direction of the $[k_u, k_v]$ vector for 256 spokes around the origin are shown in Figure 8. This sequence also produces a radially symmetric trajectory. It can be observed that the magnitude of the $[k_u, k_v]$ vector is very small around the origin, because each readout starts at the origin and traverses outward.

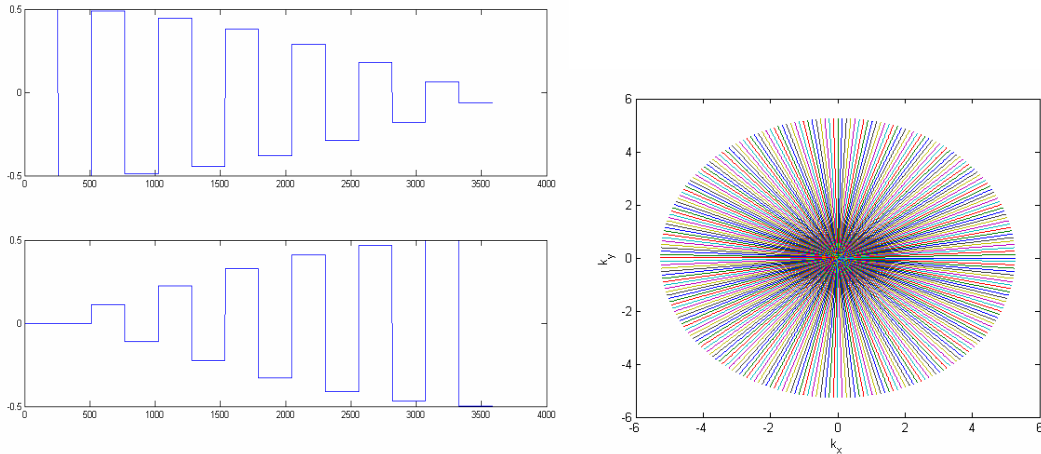


Figure 7: (a) Left: 2DPR gradient sequence for 10 angles and (b) Right: k-space trajectory for 256 radial angles.

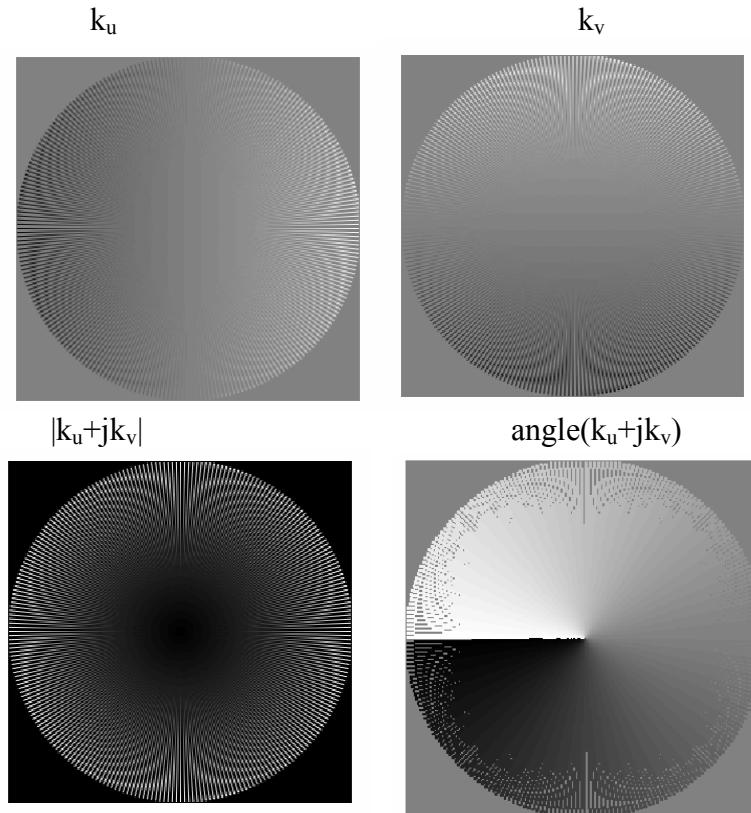


Figure 8: Velocity k-space trajectories for 2DPR sequence as a function of k_x and k_y (Bottom row) The length and direction of the $[k_u, k_v]$ vectors

The velocity k-space trajectories suggest that EPI would be more prone to flow artifacts than spiral and 2DPR, owing to the small magnitude of k_u and k_v near the origin and the circularly symmetric and smoothly varying nature of the trajectory of the latter sequences.

Simulation Results

Flow effects are studied for each of the gradient sequences mentioned above for

- (a) A 10x10 square object moving at a constant velocity
- (b) A pipe of infinite extent with plug-flow (constant velocity) at different orientations
- (c) A horizontal pipe of infinite extent with parabolic flow.

Results are presented for two different velocities, 20cm/s and 50cm/s.

1. Point Spread function for uniform velocity: Figures 9-11 show the velocity point spread function for EPI, spiral and 2DPR imaging sequences respectively for flow along x axis, y axis and along 45 degree angle. EPI produces blurring and ghosting artifacts when the flow is along readout direction. When flow is perpendicular to the readout direction, displacement and blurring is seen in the direction of flow. For flow along a 45 degree angle, displacement, blurring and ghosting artifacts are seen. Spiral sequence produces a psf that is blurred slightly in the direction of flow. The 2DPR sequence shows minimal distortion in the reconstructed image.

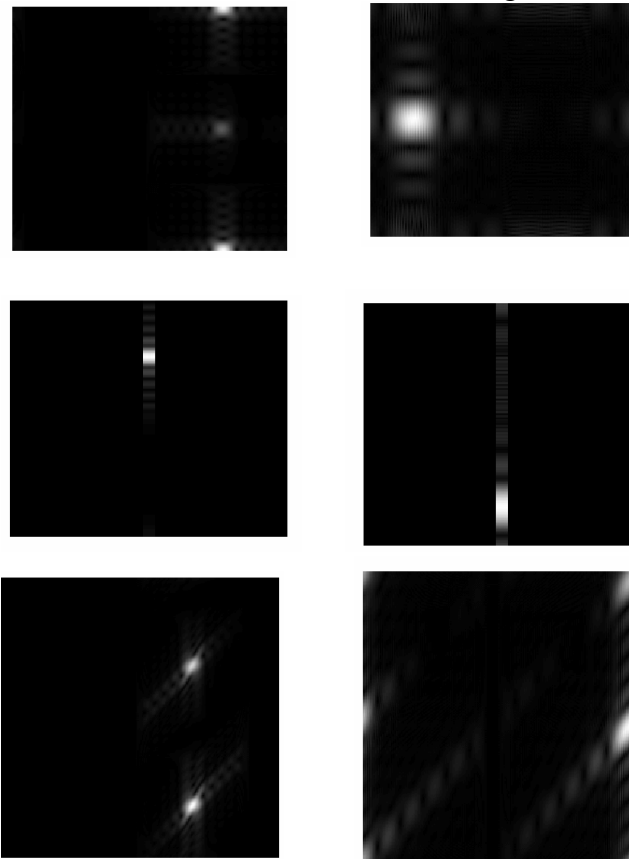


Fig.9: Velocity Point spread functions for flow in different directions for 20cm/s (left column) and 50 cms/s (right column) for EPI
 Top Row: Horizontal flow:
 Middle Row: Vertical flow
 Bottom Row: Flow at a 45 degree angle.

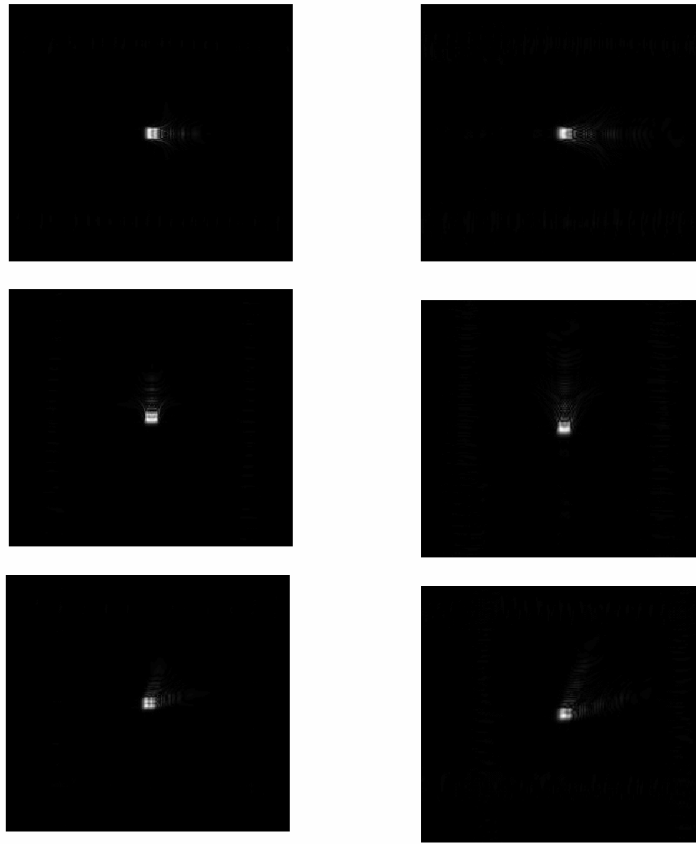


Fig.10: Velocity Point spread functions for flow in different directions for 20cm/s (left column) and 50 cms/s (right column) for spiral sequence. Top Row: Horizontal flow: Middle Row: Vertical flow Bottom Row: Flow at a 45 degree angle.

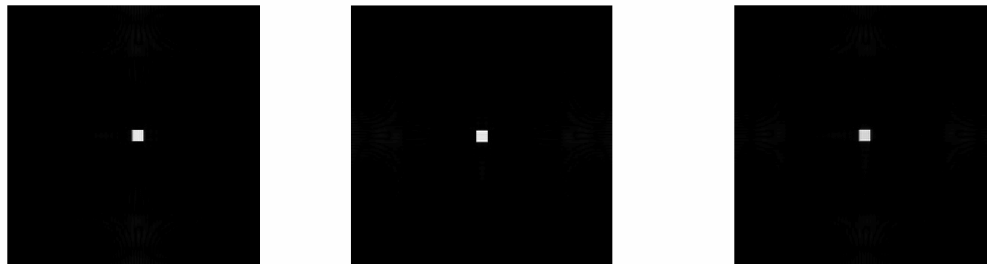


Fig.11: Velocity Point spread functions for flow in different directions for 2DPR sequence for velocity of 50cm/s. (L to R) Horizontal flow, vertical flow and flow at 45 degree angle.

2. Pipe with constant velocity flow at different orientations: A vessel with infinite extent in the direction of flow is considered. The effect of flow for EPI, spiral and 2DPR are shown in figures 12-14 respectively for a constant velocity flow of 50cm/s. EPI exhibits intensity distortions and ghosting effects, when the flow is in the readout direction. For vertical flow, no artifacts are seen and for diagonal flow, in addition to ghosting and blurring, distortion of the vessel is also observed. Spiral sequence produces slight blurring, but no significant artifacts in all three directions. 2DPR shows minimal artifacts in all three orientations.

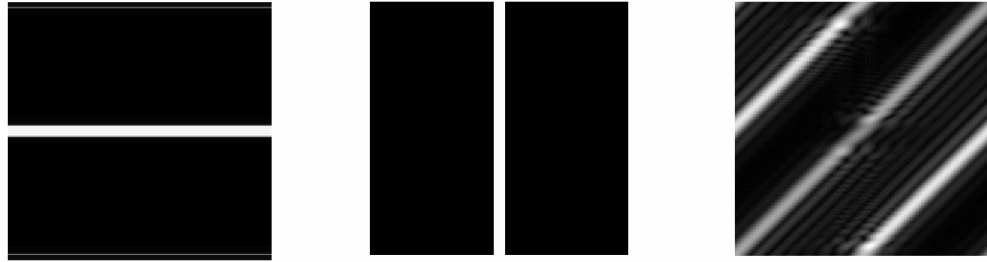


Fig.12: Constant velocity flow of 50cm/s in horizontal, vertical and 45 degree angle for EPI sequence.

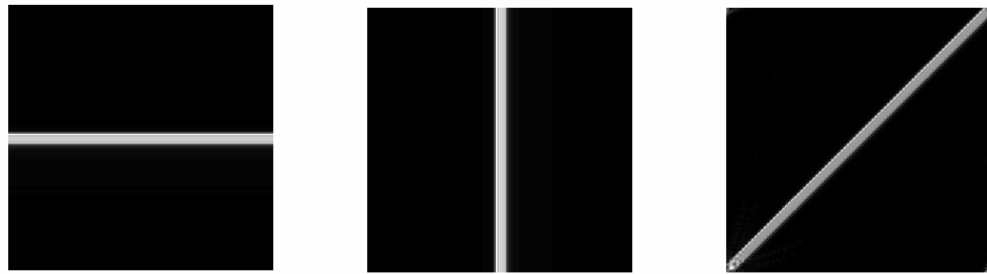


Fig.13: Constant velocity flow of 50cm/s in horizontal, vertical and 45 degree angle for spiral sequence.

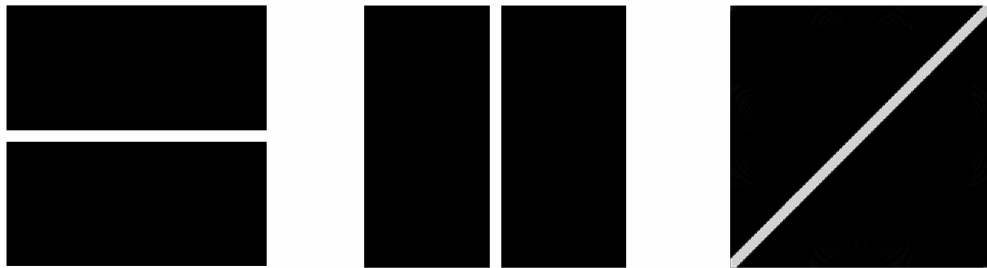


Fig.14: Constant velocity flow of 50cm/s in horizontal, vertical and 45 degree angle for 2DPR sequence for velocity of 50cm/s.

3. Parabolic flow: Parabolic flow through a horizontal pipe of infinite extent for the EPI, spiral and 2DPR sequences are shown in Figure 15. 2DPR and spiral sequences produce minimal artifacts whereas EPI produces intensity distortion and ghosting artifacts.



Fig.15: Parabolic flow of maximum velocity of 50cm/s in EPI, spiral and 2DPR sequences.

Discussion

The effect of flow is substantially different for different trajectories. This is due to the difference in the velocity k-space sampling. Spiral and 2DPR velocity space trajectories are radially symmetric with very small magnitude near the origin and in directions that point outward, away from the origin. This causes minimal phase effect and blurring in the direction of flow, which cannot be observed for the continuous stream of flow that was considered in these simulations. EPI, however exhibits discontinuities in the k_y direction and has increasing magnitude of k_u and k_v along the readout direction. Hence, it is prone to distortions due to flow. However, when the flow is perpendicular to the readout direction, no artifacts are visible because the blurring and displacement that can be seen in the point spread function, is in the direction of the flow. The diagonal orientation has the most pronounced artifacts for EPI, which can also be explained by the high magnitude of k_u and k_v in that direction.

The velocity k-space formulation of phase effects caused due to flow provides insight into the reasons for the differences in performance of various k-space trajectories. Flow artifacts can be explained by the analysis of the velocity k-space trajectory. It provides a convenient object model that incorporates velocities into the signal equation. This analysis suggests some simple possible ways of minimizing flow artifacts. For instance, using very short readout times, using radial sampling schemes instead of cartesian schemes or making the readout direction perpendicular to the direction of flow wherever it is possible. Since it is known that phase effects of flow are produced by the first moments of the gradients, designing the gradients to have zero first moment to minimize flow artifacts directly follows from the velocity k-space analysis.

References

- [1] D.G.Nishimura, P.Irarrazabal, C.H.Meyer, A Velocity k-Space Analysis of Flow Effects in Echo Planar and Spiral Imaging, *Magnetic Resonance in Medicine*, vol.33, pp.549-546, Apr. 1995.
- [2] D.G.Nishimura, Principles of Magnetic Resonance Imaging, Stanford University, 1996.

Note: The spiral gradient waveforms were provided by Taehoon Shin.