# Motion Correction with Propeller MRI: Application to Head Motion Belma Dogdas and Quanzheng Li 

## Introduction:

In this project, we implemented PROPELLER MRI (Periodically Rotated Overlapping Parallel Lines with Enhanced Reconstruction) method [1] to compensate for motion artifacts. In this method, the data is collected in concentric rectangular strips that rotate around the k space. The central portion is contained in each strip and therefore can be used to obtain a low frequency average image. The average image is used to correct for the phase, translation, rotation and scaling between strips and also to discard strips that have significant distortion by computing a correlation measure for each strip. For the reconstruction part, the 2 X gridding reconstruction scheme is used with a density estimate computed from the voronoi diagram of the k-space.

It is known that patient motion causes artifacts in MR imaging. These artifacts can be due to the tissue displacement that is caused by the patient movement between each data sampling period and the excitation RF pulse and also phase induced because of the motion through magnetic gradient fields between excitation pulse and data sampling period. Methods which collect data from center to outwards of k space such as projection reconstruction and spiral MRI reduce these motion artifacts by oversampling the central k -space which can be thought as an analogy of averaging the data in conventional imaging. Other methods try to estimate the motion or motion related phase from extra collected data which are referred as navigator echoes. [2] They generally compute the bulk transformation in the data and correct for these artifacts in the image.

The method we have implemented the PROPELLER MRI is a new technique in data collection and reconstruction. It can compensate for the phase and bulk translation, rotation and scaling in the image. Moreover it can further reject data which has significant motion based on a correlation measure. In this project, we used this method to the application of head motion.

## Methods:

The whole process of motion correction and reconstruction of the data as a flow chart is in Figure 1:


Figure 1: The Flow Chart of the Motion Correction and Reconstruction
The data we get is different from the data used in the paper. There are 16 strips each consisting of 80 parallel linear trajectories. These strips have uniform angles and inside the strips, the line of trajectories are uniformly distributed. In addition to the standard
process described in the paper we add scaling correction to model the affine transformation.

## Phase Correction:

In this step, we correct the small displacement in the k -space due to the imperfect gradient of MRI machine. The basic idea is to remove the low frequency part of the reconstructed image phase by a pyramid triangle window. The flow chart of this step is described in Figure 2.


Figure 2: The flow chart of the phase correction step
The window we used is a small window, which is as big as one strip. To make windowing easier, we rotate the each strip to a vertical position, then do the windowing. After correcting the phase in the image domain, we rotate the image using the same angle
in an opposite direction. Figure 3 gives the images of strip 1 before and after the windowing. The image is vertical because of the rotation step at the beginning. We can see that the image after windowing is much smoother because we use a narrow window to remove the high frequency component of the k-space data.


Figure 3: left, the image of strip 1 before windowing; right, the image of strip 1 after windowing

Figure 4 gives the images of strip 4 before and after the phase correction. We use the image of strip 4 because the artifact in this image is more significant. We can see that after phase correction, the artifact due to the small displacement of $k$ - space is removed.


Figure 4: left, the image of strip 4 before phase correction;
right, the image of strip 4 after phase correction
Before applying our phase correction we did some simulations to study the phase change of each step in Figure 2, and found that the windowing and the inverse 2D-FFT introduce some artifacts in the phase image. It is not surprising that all these steps in phase correction will introduce small artifacts into the image and k -space data.

## Bulk Transformation Correction:

After doing the phase correction of the data we estimated the bulk transformation of the object between each strip and corrected for these artifacts. If we restrict our model to affine case the bulk transformation are caused by rotation, translation and scaling. These transformations have nice one to one fourier transform correspondences. In our approach we estimated each transformation separately.

## Bulk Rotation Correction:

From the Fourier Transform theory we know that a rotation by $\varphi$ in image space has the same rotation $\varphi$ in its Fourier transform. (Equation 1). Therefore, using the
rotation in image space can be estimated from the rotation of the magnitude of the k space data.

$$
\begin{array}{ccc}
f(x, y) \leftrightarrow F(u, v) & \& & f\left(x^{\prime}, y^{\prime}\right) \leftrightarrow F\left(u^{\prime}, v^{\prime}\right) \\
\binom{x^{\prime}}{y^{\prime}}=\left(\begin{array}{cc}
\cos \phi & -\sin \phi \\
\sin \phi & \cos \phi
\end{array}\right)\binom{x}{y} & \& & \binom{u^{\prime}}{v^{\prime}}=\left(\begin{array}{cc}
\cos \phi & -\sin \phi \\
\sin \phi & \cos \phi
\end{array}\right)\binom{u}{v}
\end{array} \text { (Equation 1) }
$$

Since the data in PROPELLER MRI is collected by rotated strips, there exist a circular region which is collected in every strip with a diameter L/FOV. The data that falls inside this region from each strip can be used to get an average k -space data set. This data set then can be used to as a reference image to estimate the rotation in each strip. The psedocode for rotation correction:

Lets consider that R represents the Cartesian coordinates inside L/FOV diameter circle.

1. The data magnitude $M_{n}$ of each strip is gridded onto $R$.
2. Then $\mathrm{M}_{\mathrm{n}}$ 's are averaged to obtain avarage-magnitude reference data set $\mathrm{M}_{\mathrm{a}}$.
3. Each $M_{n}$ is rotated by a series of angles and gridded onto $R$.
4. Each rotated $M_{n}$ 's and $M_{a}$ are weighted by the square of its distance from k-space.
5. The correlation measure for each rotated and distance weighted $M_{n}$ and distance weighted $M_{a}$ is computed as a function of angle.
6. The angle where the correlation function is maximum is found.
7. The nearest 2 neighbors are then (total 5 points) are fitted to a second order polynomial.
8. The peak of this polynomial gives the estimated angle of rotation for the $\mathrm{n}^{\text {th }}$ strip.
9. The coordinated of the k -space are then corrected using this estimated angle of rotation.

We tested our rotation correction in the first strip of the PROPELLER MRI data. We rotated k -space by $10^{\circ}$ and try to estimate this value using the above method. The code for this simulation is Appendix. The results are as shown:
$\gg$ angle of rotation $=0.1745$
$\gg$ max_val of the correlation $=1.0000$
$\gg$ ind $=6$
$\gg$ theta(ind) $=-0.1745$
Figure 5 shows the plot of the correlation measure vs. rotation angle for the simulation data. From this figure a sharp peak is observed at angle $=-0.1745$ which is - of the exact same angle of rotation we applied to the k-space. Figure 6 shows the reconstructed images


Figure 5: Correlation measure for the simulation


Figure 6: Reconstructed images before and after correction for bulk rotation

After phase correction we applied our rotation correction algorithm to each strip in the data. Before hand we computed the magnitude average and the complex average of the strips in the circular center region which has diameter $80 / 256 \approx 0.3125$. The magnitude of the averaged reconstructed image is shown in Figure 7 which is a smooth image.


Figure 7: Reconstructed average data


Magnitude of the average of the 16 strips Afterwards we applied the rotation correction algorithm and found the angles. The resulting angles we found were very small on the order of $10^{-3}$.
angles $=1.0 \mathrm{e}-003 *\left[\begin{array}{lllllll}-0.3786 & -0.1775 & 0.1967 & 0.2262 & -0.0793 & 0.2342 & 0.3005\end{array}\right.$
$\begin{array}{lllllllll}-0.3208 & 0.2743 & -0.4609 & -0.5337 & -0.9489 & 0.0159 & 0.9778 & 0.4162 & 0.0948]\end{array}$
Then we corrected the k-space using these angles.

## Bulk Translation Correction:

From the Fourier Transform theory we know that translation in image space causes some linear phase shifts in its Fourier transform. (Equation 2). Therefore, the linear phase shift can be corrected by estimating the translation in image space.

$$
f\left(x-x_{o}, y-y_{o}\right) \leftrightarrow F(u, v) \exp \left(-2 \pi i\left[u x_{o}+v y_{o}\right]\right) \quad \text { (Equation 2) }
$$

Again as in the rotation correction part the average k-space data which is computed from the central circle of each strip is used to reconstruct the reference image. In this case however instead of using the magnitude average the complex average $k$-space data set is computed from each strip that fall inside the circular region with diameter L/FOV. The translation offsets between the reference image and the template image can be computed by convolving the reconstructed reference image and the reconstructed template image. However since convolution in image space corresponds to multiplication in k space. The translation offsets can be determined by taking the inverse Fourier transform of this product and determining the peak. Using this approach the psedocode for translation correction is:

1. The data $D_{n}$ of each strip is gridded onto $R$.
2. Then $D_{n}$ 's are averaged to obtain average reference data set $D_{a}$.
3. Compute $\mathrm{D}_{\mathrm{n}} * \mathrm{Da}^{*}$
4. IFT of the product in 4 .
5. Find the maximum of this product.
6. Fit a polynomial to the nearest neighbor points (total 3 points) in each direction.
7. The peak of this polynomial gives the estimated offset in that direction for the $\mathrm{n}^{\text {th }}$ strip.
8. The k -space data is corrected by applying the corresponding phase computed from estimated translation offsets

We tested our translation correction in the first strip of the PROPELLER MRI data. We translated our reconstructed image by 50 voxels in x direction and 10 voxels in y direction and try to estimate the bulk translation value using the above method. The code for this simulation is Appendix. The results are:
tx_c $=50$
ty_c $=10$

Figure 8 shows the plot of the correlation measure vs. translation for the simulation data. From this figure a sharp peak is observed at indices $(207,267)$ which correspond to an offset 50 in x and 10 in y direction which is the translation we applied to our data beforehand. Figure 9 shows the reconstructed images before and after translation correction.


Figure 8: Correlation measure


Figure 9: Reconstructed images before and after correction for bulk translation
After the rotation correction we applied the translation correction algorithm and found offsets in each direction. The resulting offsets we found were also very small on the order of 1 voxel.
tx $=\left[\begin{array}{llllllllllllllll}-1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0\end{array}\right]$

$$
\operatorname{ty}=\left[\begin{array}{llllllllllllllll}
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & -1 & 0 & 0
\end{array}\right]
$$

Then we corrected the k -space data by applying a linear phase that correspond to these offsets.

## Bulk Scaling Correction:

As rotation and translation in image space have corresponding properties in Fourier space, a similar relation is also observed with the scaling in the image space. Scaling in image space also causes scaling in Fourier space. However this scaling is inversely proportional such that contraction in one domain produces corresponding expansion in the other domain. (Equation 3). Therefore, the scaling factor in image space can be directly computed using the k -space data as done in rotation case.

$$
f(a x, b y) \leftrightarrow \frac{1}{|a b|} F\left(\frac{u}{a}, \frac{v}{b}\right) \quad \text { (Equation 3) }
$$

Again as in the rotation and translation correction part the average k-space data which is computed from the central circle of each strip is used to reconstruct the reference image. In this case, however the magnitude average of the complex average k space data set is computed from each strip that fall inside the circular region with diameter $\mathrm{L} / \mathrm{FOV}$ as done in rotation correction. As implemented in rotation correction the correlation measure of the scaled k -space and the average k -space is computed and the peak gives the correct scaling for that direction.

The psedocode for scaling correction:

1. The data magnitude $\mathrm{M}_{\mathrm{a}}$ that was computed for rotation correction is used as the template image.
2. Each $\mathrm{M}_{\mathrm{n}}$ is scaled by a series of scales in each direction and gridded onto R .
3. Each scaled $M_{n}$ 's and $M_{a}$ are weighted by the square of its distance from $k$-space.
4. The correlation measure for each scaled and distance weighted $\mathrm{M}_{\mathrm{n}}$ and distance weighted $M_{a}$ is computed as a function of scales.
5. The peak where the correlation function is maximum is found.
6. The nearest neighbors are then (total 3 points) are fitted to a second order polynomial in each direction as done in translation correction.
7. The peak of this polynomial gives the estimated scale for the $\mathrm{n}^{\text {th }}$ strip.
8. The coordinates of the k-space and the data are scaled accordingly.

After rotation and translation correction, we corrected the scaling. In all the strips the scaling factor is 1 which means that there does not exist a significant scaling in the image space.

## Correlation Weighting

After doing all the previous corrections, some strips will still have factors that will produce artifacts because of the significant inter-plane motion. Figure 10 shows the reconstructed images before motion correction and Figure 11 shows the image of each strip after phase correction and transformation correction. Looking more closely, we can see that there exist major artifacts in the strips through 4-8. Because poor correlation between data strips is assumed to correspond to significant through-plane motion or other factors, we can calculate a weight of each strip based on the correlation, and then multiply the data of each strip by this weight to compensate for these artifacts. The correlation measure for each strip is computed as:

$$
\begin{equation*}
x_{n}=\left|\int_{R} D_{A}^{\prime} D_{n}^{\prime *}\right| \tag{Equation4}
\end{equation*}
$$

Where $R$ is the center circle, $D_{a}{ }^{\prime}$ is the average corrected data in the center circle and $D_{n}{ }^{\prime}$ is the corrected data of the $\mathrm{n}^{\text {th }}$ strip in the center circle. The correlation weights are defined in Equation 5:

$$
\begin{equation*}
P_{n}=\left[a+(1-a)\left(\frac{x_{n}-x_{\min }}{x_{\max }-x_{\min }}\right)\right]^{p} \tag{Equation5}
\end{equation*}
$$

There are two parameters in the definition a and p, they control the trade-off between data averaging and data rejection. If the artifact in some of the strips are big, we should select small and big p , otherwise we use big a and small p . In our reconstruction, we use $\mathrm{a}=0.1$ and $\mathrm{p}=2$.

Figure 12 is the plot of correlation weight for each strip. We can see that the strips $4,5,6$, 7 and 8 have the smallest value, which is consistent with what we saw from the images in figure 11.

After we get the correlation weights, we multiply data of each strip by its corresponding weight and use combine the data for reconstruction.


Figure 10: Original reconstructed images for each strip


Figure 11: 16 reconstructed images after phase, rotation, translation and scaling correction


Figure 12: Plot of the correlation weights

## Final Reconstruction:

We used 2 X gridding for the reconstruction step. To compensate for the sample density we use the voronoin weighting instead of the weighting process described in the original paper, because it is too complicated. But there might be a slight problem by using voronion weighting in this " strip trajectory ", because it is very possible that there might be some overlaps in the position of k -space from different strips, especially in the central circle. The way to take into account of the data that is overlapping is to find the position of each overlap and average the data at that position.

Figure 13 shows the result of the final reconstructed image.


Figure 13: Final Reconstructed Image after correlation weighting the strips (Voronoin was used for density correction)

## Discussion and Conclusion:

In the phase correction, the method described in the paper works, but because the whole process involves several interpolations between image space and k space, new small artifacts are introduced into the data. Another alternative approach is to find the displacement in the k -space by computing the correlation of the k -space data of each strip to the average data of k -space in the center circle and finding the peak of the correlation measure, like we did in the translation correction.

The registration correction works very well. Looking at the original images that were reconstructed from each strip we do not see much registration errors. Therefore there have been little changes after rotation, translation and scaling corrections. Since we use head data, the movements are restricted only to rigid case. However there may occur
some artifacts due to blood motion which have to be also corrected separately. In our case we used k -space data to correct for registration errors. Alternative approach is to use the image space and determine the affine transformation using the reconstructed images. With this approach the regular registration techniques can be applied to determine the translation, rotation and scaling parameters. One of the neat methods for estimating image registration is using mutual information. Mutual information is a basic concept in information theory which measures the statistical dependence between two random variables or the amount of information that one variable contains about the other. The use of mutual information in image registration corresponds to maximizing the mutual information between the intensities of corresponding voxels of the images to be registered: when the images are aligned the amount of information they contain about each other is maximal. [5]

It is also important to select suitable values for the parameters a and p in computing the correlation weights. We can decide the parameters based on the reconstructed images of each strip after phase correction and registration correction.

In the final reconstruction, we use voronoin weighting to compensate sample density. But an alternative method should take into account the overlapping of the data in the k space as we described in reconstruction step.

## Contributions of each member in the project:

Phase Correction, Correlation weighting: Quanzheng
Rotation, Translation and Scaling Correction: Belma
Reconstruction: Belma and Quanzheng
*** Project.m is the main program, the other functions are listed in zip file.

## References:

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