Last Time …

**Linear Circuit**

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sinusoidal Steady State</strong></td>
<td></td>
</tr>
</tbody>
</table>

\[ x(t) = x_a \cos \omega t \quad \rightarrow \quad y_{ss}(t) = x_a |H| \cos(\omega t + \angle H) \]

H is Determined Using Impedances \( R, j\omega L, 1/j\omega C \)

Phasor Representation Supports KVL, KCL

Complex Power (Voltage and Current Out of Phase by \( \phi \))

\[
P = \frac{\tilde{v}\tilde{i}}{2} \cos \phi (1 + \cos 2\omega t) - \frac{\tilde{v}\tilde{i}}{2} \sin \phi \sin 2\omega t
\]

Maximum Power Transfer to \( Z_2 \) (for Thevenin \( Z_1 \)):

\[ Z_2 = Z_1^* \]
Determine $v_o(t)$

$$v_i(t) = 3 \cos 5t$$
Filter Classifications

- **Low Pass**
- **High Pass**
- **Bandpass**
- **Bandstop**

**Passive**

- \( K \leq 1 \)

**Active**

- \( K > 1 \)
Brick-Wall Low-Pass Filter

\[ H(s) = \begin{cases} K & \omega \leq \omega_o \\ 0 & \omega > \omega_o \end{cases} \]

Real Signals \[ H(s) = H(-s) \]

Inverse Transform:
\[ h(t) = \frac{1}{2\pi j} \int_{-j\infty}^{j\infty} H(s) e^{st} ds = \frac{K}{\pi t} \sin \omega_o t \]

Non-Causal
\[ h(t < 0) \neq 0 \]
\[ \omega_o = 10 \text{ rad/s} \]
\[ K = 1 \]
Realize Filters with Approximate Cutoff Characteristics

- **First Order**
  - One Capacitor or One Inductor

- **Second Order**
  - One Capacitor and One Inductor
  - Use Op-Amps to Simulate Inductor Behavior

- **Higher Order**
  - Cascade First- and Second-Order Filters
  - Tables Used to Determine Order N and Subfilter Specifications
First-Order Low Pass: Magnitude

General Transfer Characteristic: \[ H(s) = \frac{K\omega_o}{s + \omega_o} = \frac{K\omega_o}{j\omega + \omega_o} \]

Low-Frequency Asymptote: \[ |H| \approx K \]

\[ K\left|_{dB} \right. = 20 \log K \]

\( \omega = \omega_o \Rightarrow H = \frac{K}{j + 1} \Rightarrow |H| = \frac{K}{\sqrt{2}} = 0.707K = K\left|_{dB} \right. - 3\text{ dB} \]

High-Frequency Asymptote: \[ |H| \sim \frac{1}{\omega} \]
First-Order Low Pass: Phase

General Transfer Characteristic:

\[ H(s) = \frac{K\omega_o}{s + \omega_o} = \frac{K\omega_o}{j\omega + \omega_o} \]

Low Frequency

\[ \angle H(0) = 0 \]

\[ \angle H(\omega_o) = -45^\circ \]

High Frequency

\[ \angle H(\infty) = -90^\circ \]

\[ \angle H(0.1\omega_o) = -6^\circ \]

\[ \angle H(10\omega_o) = -84^\circ \]
First-Order Low Pass: Want -36 dB Attenuation at $f = 42 \text{ kHz}$  \( f_o = ? \)

$$-36 \text{ dB} \rightarrow \frac{36}{20} = 1.8$$

Decades Past $f_o$  

-20 dB / Decade

$$10^{1.8} = 63.1$$

$$f_o = \frac{42 \text{ kHz}}{63.1} = 666 \text{ Hz}$$
## Limiting Frequency Behavior

<table>
<thead>
<tr>
<th>Capacitor</th>
<th>Low Frequency</th>
<th>High Frequency</th>
</tr>
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<tbody>
<tr>
<td>$Z = \frac{1}{j\omega C}$</td>
<td>Open Circuit</td>
<td>Short Circuit</td>
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<tr>
<th>Inductor</th>
<th>Low Frequency</th>
<th>High Frequency</th>
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<tr>
<td>$Z = j\omega L$</td>
<td>Short Circuit</td>
<td>Open Circuit</td>
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</table>
Low-Pass Circuit Implementation

**RC**

\[ H = \frac{1/sC}{1/sC + R} = \frac{1/RC}{s + 1/RC} = \frac{\omega_o}{s + \omega_o} \]

**RL**

\[ H = \frac{R}{R + Ls} = \frac{R/L}{s + R/L} = \frac{\omega_o}{s + \omega_o} \]

\[ \omega_o = \frac{1}{RC} \quad f_o = \frac{1}{2\pi RC} \]

\[ \omega_o = \frac{R}{L} \quad f_o = \frac{R}{2\pi L} \]
Exercise 1

Build Me \[ R = 3.3 \, \text{k}\Omega \quad C = 3.3 \, \text{nF} \]

Measure \( f_0 \)

- Look for Constant Amplitude \( A \) at Low Frequency
- Calculate 0.707 \( A \)
- Determine the Frequency at Which Amplitude = 0.707 \( A \)
First-Order High Pass

General Transfer Characteristic: \[ H(s) = \frac{Ks}{s + \omega_0} = \frac{K\omega}{j\omega + \omega_0} \]

Magnitude (dB)

-20 0 20
-20 dB / decade

K = 10
\( \omega_0 = 1 \text{ rad/s} \)

Phase (degrees)

0 45 90
-3 dB

\( log(\omega/\omega_0) \)

\( log(\omega/\omega_0) \)
High-Pass Circuit Implementation

\[ H = \frac{R}{R + 1/sC} = \frac{s}{s + 1/RC} = \frac{s}{s + \omega_o} \]

\[ \omega_o = \frac{1}{RC} \]

\[ f_o = \frac{1}{2\pi RC} \]

\[ H = \frac{Ls}{Ls + R} = \frac{s}{s + R/L} = \frac{s}{s + \omega_o} \]

\[ \omega_o = \frac{R}{L} \]

\[ f_o = \frac{R}{2\pi L} \]
Active Low-Pass Options

\[ \omega_o = \frac{1}{RC} \]

\[ K = 1 + \frac{R_1}{R_2} \]
Active High-Pass Options

\[ \omega_0 = \frac{1}{RC} \]

\[ K = 1 + \frac{R_1}{R_2} \]

\[ \omega_0 = \frac{1}{RC} \]

\[ K = -\frac{R_1}{R} \]