First-Order Low Pass Filter

\[ H(s) = \frac{K\omega_o}{s + \omega_o} = \frac{K\omega_o}{j\omega + \omega_o} \]

-20 dB / decade

\( K = 10 \quad \omega_o = 1 \text{ rad/s} \)

\[ f_o = \frac{1}{2\pi RC} \]

First-Order High Pass Filter

\[ H(s) = \frac{Ks}{s + \omega_o} = \frac{Kj\omega}{j\omega + \omega_o} \]

-20 dB / decade

\( K = 10 \quad \omega_o = 1 \text{ rad/s} \)

\[ f_o = \frac{1}{2\pi RC} \]
Consider a First-Order High Pass Filter with $f_o = 1 \text{ MHz}$.

A Sinusoidal Output Signal has 8-dB Amplitude at $f = 10 \text{ MHz}$.

Determine the Output Signal Amplitude at $f = 32 \text{ kHz}$.

Reminder: $X_{dB} = 20 \log X$
Series RLC Filter

\[ v_R = v_i \cdot H_R(j\omega) \]

\[ H_R(j\omega) = \frac{R}{R + j\omega L + 1/j\omega C} \]

\[ = \frac{R}{R + j(\omega L - 1/\omega C)} \]

Large \( \omega \to H_R \downarrow \)

Small \( \omega \to H_R \downarrow \)

Bandpass!

\( \omega L = 1/\omega C \quad \rightarrow \quad \omega_o = \frac{1}{\sqrt{LC}} \)

In Phase

\( H_R = 1 \quad \angle H_R = 0^\circ \)

(Series) Resonant Angular Frequency
Exercise 1

Build Me

Function Generator

Measure $f_0$
Series RLC Filter (Capacitor Voltage)

\[ v_C = v_i H_C(j\omega) \]

\[ H_C(j\omega) = \frac{1/j\omega C}{R + j(\omega L - 1/\omega C)} \]

\[ H_C(j\omega_o) = -j \frac{1}{\omega_o RC} = -jQ \quad Q \angle -90^\circ \]

Quality Factor \( Q = \frac{1}{\omega_o RC} \)

Potentially Large if \( R \) is Small
Series RLC Filter (Inductor Voltage)

\[ v_L = v_i H_L(j\omega) \]

\[ H_L(j\omega) = \frac{j\omega L}{R + j(\omega L - 1/\omega C)} \]

\[ H_L(j\omega) = j \frac{\omega_o L}{R} = +jQ = Q \angle 90^\circ \]

Quality Factor \[ Q = \frac{\omega_o L}{R} \left( = \frac{1}{\omega_o RC} \right) \]

Potentially Large if R is Small
Quality Factor: Another Definition

\[ Q = \omega \frac{\text{Energy Stored}}{\text{Average Power Dissipated}} \]

At Resonance

\[ Q = \omega_o \frac{1/2 i_{max}^2 L}{1/2 i_{max}^2 R} = \frac{\omega_o L}{R} \]
Phasors for Series Resonance

\[ \vec{v}_R + \vec{v}_L + \vec{v}_C = \vec{v}_i \]

Bandpass!
Digression: Parallel Resonance

\[ H_R = \frac{i_R}{i} = \frac{1/R}{1/R + 1/j\omega L + j\omega C} = \frac{1}{1 + jR(\omega C - 1/\omega L)} \]

\[ \omega L = \frac{1}{\omega C} \quad \Rightarrow \quad \omega_o = \frac{1}{\sqrt{LC}} \]

\( H_R = 1 \quad \angle H_R = 0^\circ \)

(Parallel) Resonant Angular Frequency
Exercise 2

Build Me

![Circuit Diagram]

Function Generator

Measure B (Bandwidth)

Peak Output \( \downarrow \) by 0.707

\[ v_i \quad 50 \quad 220 \text{ pF} \quad 47 \mu\text{H} \quad 50 \quad v_R \]
General Transfer Characteristic:

\[ H = \frac{s\omega_o/Q}{s^2 + s\omega_o/Q + \omega_o^2} = \frac{j\omega_o/Q}{(\omega_o^2 - \omega^2) + (\omega\omega_o/Q)^2} \]

\[ |H| = \frac{\omega\omega_o/Q}{\sqrt{(\omega_o^2 - \omega^2)^2 + (\omega\omega_o/Q)^2}} \]

\(-3\) dB \(\Rightarrow\) \(\omega\omega_o/Q = \omega_o^2 - \omega^2\) \(\Rightarrow\) \(\omega \approx \omega_o \pm \omega_o/2Q\)

Bandwidth \(B = \frac{\omega_o}{Q}\) Make \(Q\) Large for Filter Selectivity
Second-Order Bandpass Filter

Magnitude (dB)

Increasing Q

Q = 1, 1/\sqrt{2}, 1, 2

Phase (degrees)

20 dB / decade

K = 10
\omega_o = 1 \text{ rad/s}

Increasing Q
**Exercise 3**

**Build Me**

![Circuit Diagram]

- Function Generator
- Observe Inductor Voltage
- Observe Capacitor Voltage
- Filter Types?

**Be Careful With Grounds**
Second-Order Lowpass Filter

General Transfer Characteristic:

\[
H = \frac{\omega_0^2}{s^2 + s\omega_0/Q + \omega_0^2}
\]

Maximally “Flat” for \( Q = 1/\sqrt{2} = 0.707 \)
General Transfer Characteristic:

\[ H = \frac{s^2}{s^2 + s\omega_o/Q + \omega_o^2} \]

Maximally “Flat” for \( Q = 1/\sqrt{2} = 0.707 \)